Efficient and reliable road modeling for digital maps based on cardinal spline

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Abstract: In order to realize an optimal balance between the efficiency and reliability requirements of road models, a road modeling method for digital maps based on cardinal spline is studied. First, the cardinal spline is chosen to establish an initial road model, which is specified by a series of control points and tension parameters. Then, in view of the initial road model, a gradual optimization algorithm, which can determine the reasonable control points and optimal tension parameters according to the degree of the change of road curvature, is proposed to determine the final road model. Finally, the proposed road modeling method is verified and evaluated through experiments, and it is compared with the conventional method for digital maps based on the B-spline. The results show that the proposed method can realize a near-optimal balance between the efficiency and reliability requirements. Compared with the conventional method based on the B-spline, this method occupies less data storage and achieves higher accuracy.

Key words: cardinal spline; digital map; road modeling; gradual optimization; optimal balance

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In recent years, there has been a significant amount of progress in developing intelligent transportation system (ITS) applications such as vehicle navigation systems, intelligent cruise control systems and advanced driver assistance systems[1−3]. These applications require detailed road environment information to handle the challenges of the complex traffic scenarios. A digital map can provide detailed information of the road environment for these applications. Therefore, efficient and reliable digital maps are crucial to further the development of ITS applica-

tions[4−6]. Road modeling is the vital part of generating digital maps. From a technical point of view, a satisfactory road model for digital maps should realize an optimal balance between the following two requirements: efficiency and reliability. First, the efficiency requirement is relevant to the data storage of the road model. A road model should have an efficient and compact data structure to reduce the data storage such that the road model can effectively transfer road information via wireless networks. Secondly, the reliability requirement is relevant to the quality of the road model. A road model should contain the road geometry information with the desired accuracy and integrity[1,3,5].

It is a consensus that the efficiency requirement and the reliability requirement are contradictory during the process of road modeling. The tradeoff for the increased accuracy of a road model is the cost of increased data storage. Therefore, it is difficult for a road model to satisfy the aforementioned two requirements simultaneously[1,5−7]. The previous studies considered the reliability requirement excessively while ignoring the efficiency requirement during the process of road modeling. In order to balance these two requirements well, it is better to select an appropriate spline curve to build a road model. Currently, various spline curves were widely selected to build road models in previous literature[1,3,7−11]. For example, the Hermite spline curve is a representative interpolating spline curve and the Hermite spline-based road model can be locally adjusted. However, the Hermite spline curve has a disadvantage that large amounts of data points need to be stored and processed. Therefore, it is difficult for the Hermite spline model to comply with the efficiency requirement. Moreover, the B-spline curve is a representative approximating a spline curve and the B-spline-based road model has an advantage that the change of local control points do not affect the entire shape of the road model, which makes local modification of a road model possible. However, it is difficult to extract road geometry information and the calculation of the first and second derivatives of the function is complicated. Therefore, the availability of the B-spline-based road model is poor.

In this paper, a probe vehicle equipped with a single-enclosure GPS + INS positioning system measures the raw roadway geometry data and acquired data will be processed manually to remove the outliers. Then, the cardinal spline is selected to build an initial road model. The initial road model is specified by a series of control points and tension parameters. In view of the initial road model, a gradual optimization algorithm is proposed to determine reasonable control points and optimal tension parameters according to the degree of the change of road curvature. Selection of reasonable control points can improve data storage efficiency and optimal tension parameters can satisfy a desired accuracy. Consequently, the road modeling method proposed in this paper realizes a near-optimal balance between the efficiency and the reliability requirements.

1   Road Modeling

Currently, a spline curve can generally be divided into two categories: the approximating spline and interpolating spline. The cardinal spline is a family of interpolating spline. To the best of the authors' knowledge, the cardinal spline has been researched for a long time in the field of the CAD/CAM industry and the cardinal spline representation of the road geometry is rarely applied in the field of road modeling in the previous literature[12-13]. In order to realize a near-optimal balance between the efficiency and reliability requirements, the cardinal spline is selected to build the initial road model and a gradual optimization algorithm is developed to determine reasonable control points and optimal tension parameters of this road model.

1.1   Cardinal spline road model

The processed data points of the roadway geometry from a probe vehicle equipped with a single-enclosure GPS + INS positioning system are used to build the cardinal spline road model. The cardinal spline road model is a sequence of individual piecewise cubic curves joined to form a larger curve, and each cardinal spline segment is defined by four control points P_{i-1}, P_i, P_{i+1}, P_{i+2}, with the curve drawn only from P_i to P_{i+1}. The tangent at each control point P_i is calculated using the previous and next control points on the spline, without being given by humans. A single cardinal spline segment can be completely determined by four consecutive control points, in which the middle two control points are the endpoints of the cardinal spline segment and the other two control points are used to calculate the tangents at the endpoints of this cardinal spline segment. Let us suppose that P_{i-1}, P_i, P_{i+1}, P_{i+2} are four given consecutive control points (each of them has x and y values) and $P(u)$ is a parametric cubic polynomial between the control points P_i and P_{i+1} (u is the parameter). Hence, the boundary condition for establishing a single cardinal spline segment using four points P_{i-1} to P_{i+2} is

$$\left.\begin{array}{l} P(0) = P_i \\ P(1) = P_{i+1} \\ P'(0) = (1-t)(P_{i+1} - P_{i-1}) \\ P'(1) = (1-t)(P_{i+2} - P_i) \end{array}\right\} \quad (1)$$

where $P(0)$ and $P(1)$ are the position vectors of $P(u)$ at the two endpoints of the spline segment between P_i and P_{i+1}, respectively; $P'(0)$ and $P'(1)$ are the tangent vectors of $P(u)$ at the two endpoints of the spline segment between P_i and P_{i+1}, respectively; and t is a tension parameter which affects the tightness of the cardinal spline road model.

Additionally, u is the parameter of the spline segment and ranges from 0 to 1 between P_i and P_{i+1}(see Fig. 1).
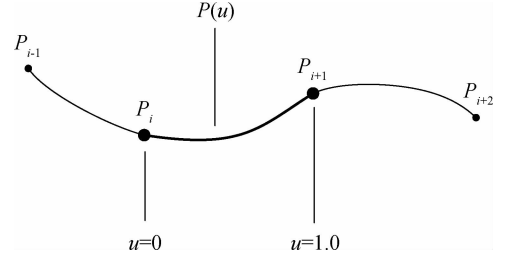

Fig. 1   The spline segment when u ranges from 0 to 1 between P_i and P_{i+1}

Eq. (1) is converted into a matrix form, as follows:

$$P(u) = [\, u^3 \quad u^2 \quad u \quad 1\,] M_{\mathrm{c}} \begin{bmatrix} P_{i-1} \\ P_i \\ P_{i+1} \\ P_{i+2} \end{bmatrix} \quad (2)$$

$$M_{\mathrm{c}} = \begin{bmatrix} -s & 2-s & s-2 & s \\ 2s & s-3 & 3-2s & -s \\ -s & 0 & s & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (3)$$

where $s = 1 - t/2$.

Eq. (2) is expanded into a polynomial form:

$$\begin{aligned} P(u) = {} & P_{i-1}(-su^3 + 2su^2 - su) + \\ & P_i[(2-s)u^3 + (s-3)u^2 + 1] + \\ & P_{i+1}[(s-2)u^3 + (3-2s)u^2 + su] + \\ & P_{i+2}(su^3 - su^2) \end{aligned} \quad (4)$$

Now, we can decompose Eq. (4) into the components of the x and y directions in the two-dimensional plane, and obtain the following expression of the cubic polynomial of the spline segment between the control points P_i and P_{i+1} in the x and y directions:

$$x(u) = [\,u^3 \quad u^2 \quad u \quad 1\,] \begin{bmatrix} -s & 2-s & s-2 & s \\ 2s & s-3 & 3-2s & -s \\ -s & 0 & s & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{i-1} \\ x_i \\ x_{i+1} \\ x_{i+2} \end{bmatrix}$$

$$y(u) = [\,u^3 \quad u^2 \quad u \quad 1\,] \begin{bmatrix} -s & 2-s & s-2 & s \\ 2s & s-3 & 3-2s & -s \\ -s & 0 & s & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_{i-1} \\ y_i \\ y_{i+1} \\ y_{i+2} \end{bmatrix}$$

$$(5)$$

where $0 \leqslant u \leqslant 1$.

Then, after the expansion of Eq. (5), the cardinal spline segment between the control points P_i and P_{i+1} is given as

$$\left. \begin{array}{l} x(u) = A_0^i + A_1^i u + A_2^i u^2 + A_3^i u^3 \\ y(u) = B_0^i + B_1^i u + B_2^i u^2 + B_3^i u^3 \end{array} \right\} \qquad (6)$$

where

$$A_0^i = x_i$$
$$A_1^i = -sx_{i-1} + sx_{i+1}$$
$$A_2^i = 2sx_{i-1} + (s-3)x_i + (3-2s)x_{i+1} - sx_{i+2}$$
$$A_3^i = -sx_{i-1} + (2-s)x_i + (s-2)x_{i+1} + sx_{i+2}$$
$$B_0^i = y_i$$
$$B_1^i = -sy_{i-1} + sy_{i+1}$$
$$B_2^i = 2sy_{i-1} + (s-3)y_i + (3-2s)y_{i+1} - sy_{i+2}$$
$$B_3^i = -sy_{i-1} + (2-s)y_i + (s-2)y_{i+1} + sy_{i+2}$$

Thus, it can be seen that each cardinal spline segment is specified by four given consecutive control points and a tension parameter which is determined according to the control requirements and accuracy requirements of the actual application scenarios. The cardinal spline road model with different values for the tension parameter will produce different curves through a given series of control points. The cardinal spline road model with four different values for the tension parameter passes through the same series of control points, as shown in Fig. 2.
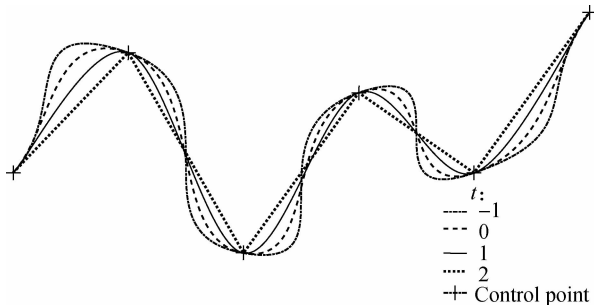

t:
- - - - −1
- - - 0
—— 1
⋯⋯ 2
+ Control point

Fig. 2 The cardinal spline road model with four different values for the tension parameter

Note that the cardinal spline road models in Fig. 2 share the same tangent line at the starting point, which is the line drawn from the starting point to the next point along the curve. Likewise, the shared tangent at the ending point is the line drawn from the ending point to the previous point on the curve. The tangent line at other points is parallel to the line drawn from the previous point to the next point along the curve.

1.2 Gradual optimization algorithm for road modeling

The input of this algorithm is the set of the processed trajectory data points of the probe vehicle, and the output is the near-optimal cardinal spline road model, which can realize a near-optimal balance between the two requirements of efficiency and reliability. A gradual optimization algorithm for road modeling consists of two main steps: The selection of control points and the adjustment of tension parameters.

In the first step, due to the fact that the curvature of urban roads varies greatly, we need to select appropriate points from a series of data points as the control points of the cardinal spline road model according to the degree of the change of road curvature. Due to the useful features of the cardinal spline such as flexibility and local modification, the distribution of control points should be dense in the road section with the large curvature change and the distribution of control points can be properly sparse in the road section with the small curvature change, which can give consideration to the reliability and efficiency requirements. The curvature is calculated as

$$C = \frac{|y''|}{(1 + (y')^2)^{3/2}} \qquad (7)$$

where C is the curvature value; y' is the first derivative of the fitted curve; y'' is the second derivative of the fitted curve.

It is shown from Eq. (7) that the focus of the curvature is to obtain the first and second derivatives of the fitted curve. Based on this, we first perform natural cubic spline interpolation of sequential data points in order to obtain the first and second derivatives of the interpolation at each data point. Furthermore, the curvature value of each data point is calculated by Eq. (7). During this step, we select control points for the cardinal spline road model by comparing the curvature values of two adjacent data points. The threshold of curvature change is artificially preset according to the actual application. If the threshold value is too small, the data volume of the cardinal spline road model will not decrease obviously, and too large a value will lead to an incomplete road shape and loss of precision. In this paper, the threshold value is set to be 0.01.

In the second step, the value of the tension parameter for each cardinal spline segment which is specified by a set of control points from the previous step are initially set

to be −1. Then, the tension parameter of each cardinal spline segment is gradually adjusted independently by minimizing the error between the given data points and the cardinal spline segment. Generally, the adjustment range of tension parameters is between −1 and 2 in this paper. The least-square method is applied to find optimal tension parameters.

2 Experimental Results

2.1 Experiment setup

To evaluate the performance of the proposed road modeling method, experiments are carried out on a Chery TIGGO5 SUV vehicle. The Olympic Sports Center of Nanjing City is selected as the experimental site. The raw road geometry data was collected using the probe vehicle equipped with a GPS + INS vehicle positioning system (NovAtel SPAN-CPT system). NovAtel SPAN-CPT system is an accurate and reliable GPS + INS vehicle positioning system, which can provide accurate positioning information via post-processing even under adverse environments[14]. In this paper, the probe vehicle was driven at a stationary driving speed of 40 to 60 km/h along the centerline of the lane.

2.2 Road modeling

We select appropriate control points for the cardinal spline road model from sequential obtained raw data points according to the change of curvature value. First, we perform natural cubic spline interpolation of sequential data points acquired from the trajectory of the probe vehicle in order to obtain the first and second derivatives of the interpolation at each data point. Then, the curvature value of each data point is calculated by Eq. (7). The threshold value of curvature change in the gradual optimization algorithm is set to be 0.01 with an overall consideration of the efficiency and reliability requirements in this paper. Then, we set the initial value of the tension parameter for each cardinal spline segment to be −1. The tension parameter for each cardinal spline segment is adjusted independently by minimizing the error between the given data points and the cardinal spline segment. The adjustment range of tension parameters is between −1 and 2 in this paper. Fig. 3 shows the process of road modeling.

2.3 Performance evaluation of road modeling

The 378 raw position data points are used for the inputs of the cardinal spline road model based on the gradual optimization algorithm. After the selection of control points and the adjustment of the tension parameter, the near-optimal cardinal spline road model is achieved, which contains 193 control points. There are 192 individual cardinal spline segments in this experiment. To evaluate the performance of the proposed road modeling method, we conduct
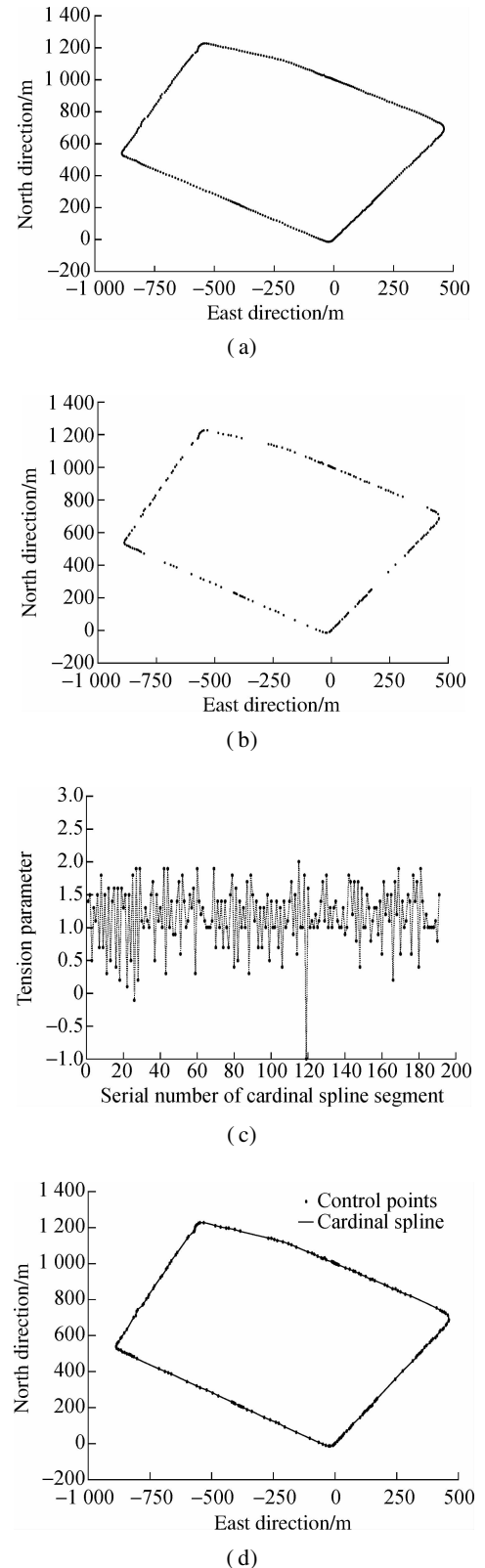

Fig.3 The process of road modeling. (a) Raw data points from the trajectory of the probe vehicle; (b) Selection result of control points; (c) Adjustment result of tension parameters; (d) Result of road modeling

a comparison with a B-spline road model proposed in Ref.[1]. The B-spline road model has been proven to outperform various previous road modeling methods, including methods using polygons, clothoids and partial

types of spline curves. The specific comparison results are presented in Tab. 1 and Tab. 2.

Tab. 1   Efficiency comparison of two methods

Parameters	B-spline	Cardinal spline
Reliability	High	High
Globalerror RMS/m	1. 25	1. 15
Efficiency	Middle	High
Data storage	1 142	578
Flexibility	Middle	High

Tab. 2   Reliability comparison of two methods

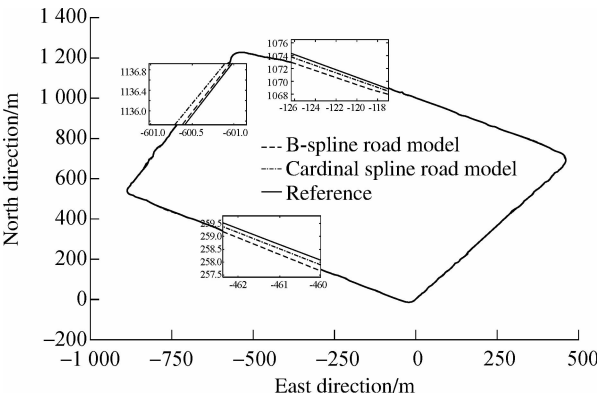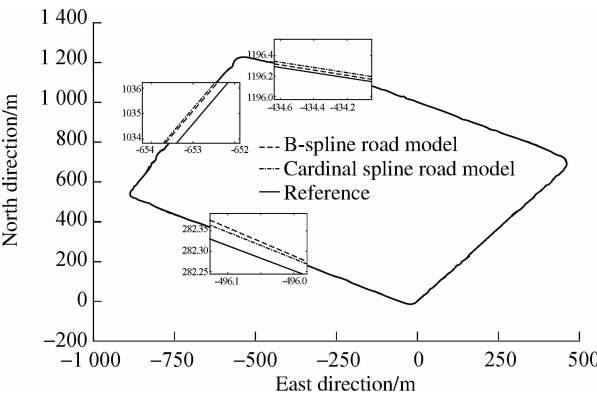| Parameters	B-spline	Cardinal spline
Reliability	Low	High
Globalerror RMS/m	2. 83	1. 15
Efficiency	High	High
Data storage	578	578
Flexibility	Middle	High

Fig. 4   Efficiency comparison of two methods

Fig. 5   Reliability comparison of two methods

From the comparison results above, the B-spline road modeling method achieves the global accuracy of 1. 25 m by using 1 142 floating-point numbers and achieves the global accuracy of 2. 83 m by using 578 floating-point numbers. Meanwhile, the cardinal spline road modeling method proposed in this paper achieves the global accuracy of 1. 25 m by using 578 floating-point numbers. Fig. 4 shows the efficiency comparison of two methods. Results show that the cardinal spline road modeling method uses fewer data points to meet a desired global accuracy than

the B-spline road modeling method does. Fig. 5 shows the reliability comparison of two methods. Results show that the cardinal spline road modeling method achieves a better global accuracy than the B-spline road modeling method does in the case of using the same number of data points. It is obvious that the cardinal spline road modeling method outperforms the B-spline road modeling method. In general, this method commendably realizes a near-optimal balance between the efficiency and reliability requirements.

3   Conclusion

In this paper, we have studied a road modeling method for digital maps based on the cardinal spline that can balance the efficiency and reliability requirements. A probe vehicle equipped with a single-enclosure GPS + INS positioning system measured the raw roadway geometry data. Also, a gradual optimization algorithm is presented to determine reasonable control points from the raw data points according to the degree of the change of road curvature. The gradual optimization algorithm can ensure that the near-minimum number of the control points and optimal tension parameters for the cardinal spline road model are selected to achieve desired accuracy. The experiments demonstrate that compared with the conventional method based on B-spline, the proposed road modeling method occupies less data storage and achieves higher accuracy.

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基于 Cardinal 样条的数字地图道路建模方法

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摘要:为了实现道路模型高效性与可靠性之间的最优平衡,研究了一种基于 Cardinal 样条的数字地图道路建模方法. 首先,使用 Cardinal 样条建立一个初始的道路模型,此模型由一系列的控制点和张力参数确定;然后,针对该初始模型,提出一种渐进优化算法确定最终的道路模型,该算法根据道路曲率的变化程度确定合理的控制点和最优的张力参数. 最后,通过实验对该道路建模方法进行验证和评估,并将此方法与基于 B 样条的数字地图道路建模方法进行对比分析. 结果表明,所提方法可较好地实现道路模型高效性与可靠性之间的平衡;相比于 B 样条道路建模方法,该方法所占用的数据存储量更小,所达到的精度更高.

关键词: Cardinal 样条;数字地图;道路建模;渐进优化;最优平衡

中图分类号:U411