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Generalized Cayley-Hamilton theorem for core-EP inverse matrix and DMP inverse matrix

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Abstract: By using the classical Cayley-Hamilton theorem, the polynomial equations of the core-EP inverse matrix and Drazin-Moore-Penrose (DMP) inverse matrix are given, respectively. If the characteristic polynomial of the singular matrix $\boldsymbol{A}$, $p_A(s)=\det(s\boldsymbol{E}_n-\boldsymbol{A})=s^n+a_{n-1}s_{n-1}+\ldots+a_1 s$, is given, then $f_A(\boldsymbol{A}^{\oplus})=\boldsymbol{0}$ and $f_A(\boldsymbol{A}^{\mathrm{d},+})=\boldsymbol{0}$ in which $f_A(\boldsymbol{A})=a_1x^n+a_2x^{n-1}+\ldots+a_{n-1}x^2+x$, and $\boldsymbol{A}^{\oplus}$ and $\boldsymbol{A}^{\mathrm{d},+}$ are the core-EP inverse and the DMP inverse of $\boldsymbol{A}$, respectively. Furthermore, some properties of the characteristic polynomials of $\boldsymbol{A}^{\mathrm{D}}\in\boldsymbol{C}_{n,n}$ and $\boldsymbol{A}^{\oplus}\in\boldsymbol{C}_{n,n}$ are derived.

Key words: Cayley-Hamilton theorem; characteristic equation; Drazin inverse; Drazin-Moore-Penrose (DMP) inverse; core-EP inverse

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In this paper, we use the following notations. The symbol $\boldsymbol{C}_{m,n}$ is the set of $m\times n$ matrices with complex entries, and $\mathrm{rk}(\boldsymbol{A})$ represents the rank of $\boldsymbol{A}\in\boldsymbol{C}_{m,n}$. Let $\boldsymbol{A}\in\boldsymbol{C}_{n,n}$, and then the smallest non-negative integer k, which satisfies $\mathrm{rk}(\boldsymbol{A}^{k+1})=\mathrm{rk}(\boldsymbol{A}^k)$, is called the index of $\boldsymbol{A}$ and is denoted as $\mathrm{Ind}(\boldsymbol{A})$. The Moore-Penrose inverse of $\boldsymbol{A}\in\boldsymbol{C}_{m,n}$ is defined as the unique matrix $\boldsymbol{X}\in\boldsymbol{C}_{n,m}$ satisfying the equations $\boldsymbol{AXA}=\boldsymbol{A}$, $\boldsymbol{XAX}=\boldsymbol{X}$, $(\boldsymbol{AX})^*=\boldsymbol{AX}$, $(\boldsymbol{XA})^*=\boldsymbol{XA}$, and is denoted as $\boldsymbol{X}=\boldsymbol{A}^+$. The Drazin inverse of $\boldsymbol{A}\in\boldsymbol{C}_{n,n}$ is defined as the unique matrix $\boldsymbol{X}\in\boldsymbol{C}_{n,n}$ satisfying the equations $\boldsymbol{XA}^{k+1}=\boldsymbol{A}^k$, $\boldsymbol{XAX}=\boldsymbol{X}$, $\boldsymbol{AX}=\boldsymbol{XA}$ and is usually denoted as $\boldsymbol{X}=\boldsymbol{A}^{\mathrm{D}}$ (see Ref. [1]). The core-EP inverse of $\boldsymbol{A}\in\boldsymbol{C}_{n,n}$ is defined as the unique matrix $\boldsymbol{X}\in\boldsymbol{C}_{n,n}$ satisfying the equations $\boldsymbol{XA}^{k+1}=\boldsymbol{A}^k$, $\boldsymbol{XAX}=\boldsymbol{X}$ and $(\boldsymbol{AX})^*=\boldsymbol{AX}$, and is denoted as $\boldsymbol{X}=\boldsymbol{A}^{\oplus}$[2]. The DMP inverse of $\boldsymbol{A}\in\boldsymbol{C}_{n,n}$ is defined as the unique matrix $\boldsymbol{X}\in\boldsymbol{C}_{n,n}$ satisfying the equations $\boldsymbol{XAX}=\boldsymbol{X}$, $\boldsymbol{XA}=\boldsymbol{A}^{\mathrm{D}}\boldsymbol{A}$ and $\boldsymbol{A}^m\boldsymbol{X}=\boldsymbol{A}^k\boldsymbol{A}^+$, and is denoted as $\boldsymbol{X}=\boldsymbol{A}^{\mathrm{d},+}$[3]. More details of the Drazin, core-EP, DMP inverses can be seen in Refs. [4 − 8].

The Cayley-Hamilton theorem has many applications in nonlinear time-varying systems, electric circuits, etc. The classical Cayley-Hamilton theorem was extended to the fractional continuous-time and discrete-time linear systems[9], nonlinear time-varying systems with square and rectangular systems[10], the Drazin inverse matrix and standard inverse matrix[11], etc. More details about the Cayley-Hamilton theorem and its applications can be read in Refs. [9 − 13]. Therefore, it is very interesting to investigate the Cayley-Hamilton theorem for the core-EP inverse matrix and DMP inverse matrix. In this paper, our main tools are core-EP decomposition and generalized inverses.

1 Preliminaries

In this section, we present some preliminary results.

Theorem 1[14, Cayley-Hamilton theorem] Let $p_A(s)=\det(s\boldsymbol{E}_n-\boldsymbol{A})$ be the characteristic polynomial of $\boldsymbol{X}\in\boldsymbol{C}_{n,n}$. Then $p_A(\boldsymbol{A})=\boldsymbol{0}$.

Theorem 2[14] Let $\boldsymbol{A}\in\boldsymbol{C}_{n,n}$ is singular, i.e. $\det(\boldsymbol{A})=0$, and the characteristic polynomial of $\boldsymbol{A}$ be

$$p_A(s)=\det(s\boldsymbol{E}_n-\boldsymbol{A})=s^n+a_{n-1}s_{n-1}+\ldots+a_1 s \tag{1}$$

Then

$$f_A(\boldsymbol{A}^{\mathrm{D}})=a_1(\boldsymbol{A}^{\mathrm{D}})^n+a_2(\boldsymbol{A}^{\mathrm{D}})^{n-1}+\ldots+a_{n-1}(\boldsymbol{A}^{\mathrm{D}})^2+\boldsymbol{A}^{\mathrm{D}}=\boldsymbol{0} \tag{2}$$

Lemma 1[15, core-nilpotent decomposition] Let $\boldsymbol{A}\in\boldsymbol{C}_{n,n}$ be with $\mathrm{Ind}(\boldsymbol{A})=k$. Then $\boldsymbol{A}$ can be written as the sum of matrices $\hat{\boldsymbol{A}}_1$ and $\hat{\boldsymbol{A}}_2$, i.e. $\boldsymbol{A}=\hat{\boldsymbol{A}}_1+\hat{\boldsymbol{A}}_2$ where $\mathrm{Ind}(\hat{\boldsymbol{A}}_1)\leqslant 1$, $\hat{\boldsymbol{A}}_2$ is nilpotent, and $\hat{\boldsymbol{A}}_2\hat{\boldsymbol{A}}_1=\hat{\boldsymbol{A}}_1\hat{\boldsymbol{A}}_2=\boldsymbol{0}$. Here one or both of $\hat{\boldsymbol{A}}_1$ and $\hat{\boldsymbol{A}}_2$ can be null. Furthermore, there is a nonsingular matrix $\boldsymbol{P}$ such that

$$\hat{\boldsymbol{A}}_1=\boldsymbol{P}\begin{bmatrix}\boldsymbol{\Sigma} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{0}\end{bmatrix}\boldsymbol{P}^{-1},\ \hat{\boldsymbol{A}}_2=\boldsymbol{P}\begin{bmatrix}\boldsymbol{0} & \boldsymbol{0}\\ \boldsymbol{0} & \overline{\boldsymbol{N}}\end{bmatrix}\boldsymbol{P}^{-1} \tag{3}$$

where $\boldsymbol{\Sigma}\in\boldsymbol{C}_{\mathrm{rk}(\boldsymbol{A}),\mathrm{rk}(\boldsymbol{A})}$ is non-singular, and $\overline{\boldsymbol{N}}$ is nilpotent and $\overline{\boldsymbol{N}}^k=\boldsymbol{0}$.

Lemma 2[15, core-EP decomposition] Let $\boldsymbol{A}\in\boldsymbol{C}_{n,n}$ be with $\mathrm{Ind}(\boldsymbol{A})=k$. Then $\boldsymbol{A}$ can be written as the sum of matrices $\boldsymbol{A}_1$ and $\boldsymbol{A}_2$, i.e. $\mathrm{A}=\boldsymbol{A}_1+\boldsymbol{A}_2$ where $\mathrm{Ind}(\boldsymbol{A}_1)\leqslant 1$, $\boldsymbol{A}_2$ is nilpotent, and $\boldsymbol{A}_1^*\boldsymbol{A}_2=\boldsymbol{A}_2\boldsymbol{A}_1=\boldsymbol{0}$. Here one or both of $\boldsymbol{A}_1$

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and $\boldsymbol{A}_2$ can be null. Furthermore, there is a unitary matrix $\boldsymbol{U}$ such that

$$\boldsymbol{A}_1 = \boldsymbol{U}\begin{bmatrix} \boldsymbol{T} & \boldsymbol{S} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\boldsymbol{U}^*,\ \boldsymbol{A}_2 = \boldsymbol{U}\begin{bmatrix} \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{N} \end{bmatrix}\boldsymbol{U}^* \tag{4}$$

where $\boldsymbol{T} \in \boldsymbol{C}_{\mathrm{rk}(\boldsymbol{A}),\,\mathrm{rk}(\boldsymbol{A})}$ is non-singular, $\boldsymbol{N}$ is nilpotent and $\boldsymbol{N}^k = \boldsymbol{0}$.

Lemma 3 Let the core-EP decomposition of $\boldsymbol{A} \in \boldsymbol{C}_{n,n}$ be as in Lemma 2. Then the core-EP inverse of $\boldsymbol{A}$ is

$$\boldsymbol{A}^{\oplus} = \boldsymbol{U}\begin{bmatrix} \boldsymbol{T}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\boldsymbol{U}^* \tag{5}$$

Let the core-EP decomposition of $\boldsymbol{A}$ be as in (4). Then

$$\boldsymbol{A}^p = \boldsymbol{U}\begin{bmatrix} \boldsymbol{T}^p & \boldsymbol{\Phi}_p \\ \boldsymbol{0} & \boldsymbol{N}^p \end{bmatrix}\boldsymbol{U}^* \tag{6}$$

and

$$(\boldsymbol{A}^{\oplus})^p = \boldsymbol{U}\begin{bmatrix} \boldsymbol{T}^{-p} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\boldsymbol{U}^* \tag{7}$$

where $\boldsymbol{\Phi}_p = \sum_{i=1}^{p} \boldsymbol{T}^{i-1}\boldsymbol{S}\boldsymbol{N}^{p-i}$. It is easy to confirm that $\boldsymbol{\Phi}_j = T^{j-k}\boldsymbol{\Phi}_k$, where $j \geqslant k$.

2 Main Results

In this section the classical Cayley-Hamilton theorem will be extended to the core-EP inverse matrix and DMP inverse matrix. By assumption, matrix $\boldsymbol{A}$ is singular, i. e. $\det(\boldsymbol{A}) = 0$.

Lemma 4 Let the characteristic polynomial of $\boldsymbol{A} \in \boldsymbol{C}_{n,n}$ be as in Eq. (1). Then

$$f_A(\boldsymbol{A}^{\oplus}) = a_1(\boldsymbol{A}^{\oplus})^n + a_2(\boldsymbol{A}^{\oplus})^{n-1} + \ldots + a_{n-1}(\boldsymbol{A}^{\oplus})^2 + \boldsymbol{A}^{\oplus} = \boldsymbol{0} \tag{8}$$

Proof Using (1) and Theorem 1 we obtain

$$\boldsymbol{A}^n + a_{n-1}\boldsymbol{A}^{n-1} + \ldots + a_1\boldsymbol{A} = \boldsymbol{0} \tag{9}$$

It follows from Lemma 2 and (6) that

$$\boldsymbol{U}\begin{bmatrix} \boldsymbol{T}^n + \sum_{p=1}^{n-1} a_p\boldsymbol{T}^p & * \\ \boldsymbol{0} & \Delta \end{bmatrix}\boldsymbol{U}^* = \boldsymbol{0} \tag{10}$$

Post-multiplying (10) by

$$\boldsymbol{U}\begin{bmatrix} \boldsymbol{T}^{-(n+1)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\boldsymbol{U}^* = \boldsymbol{0}$$

we have

$$\boldsymbol{U}\begin{bmatrix} \boldsymbol{T}^{-1} + \sum_{p=1}^{n-1} a_p\boldsymbol{T}^{p-(n+1)} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}\boldsymbol{U}^* = \boldsymbol{0} \tag{11}$$

Therefore, by applying (7), we obtain (8).

Example 1 Let

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Then Ind $(\boldsymbol{A}) = 2$, the Drazin $\boldsymbol{A}^{\mathrm{D}}$ is

$$\boldsymbol{A}^{\mathrm{D}} = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

and the core-EP inverse $\boldsymbol{A}^{\oplus}$ is

$$\boldsymbol{A}^{\oplus} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

The characteristic polynomial of $\boldsymbol{A}$ is

$$\det(s\boldsymbol{E}_4 - \boldsymbol{A}) = \begin{vmatrix} s-1 & 0 & -1 & 0 \\ 0 & s-1 & 0 & -1 \\ 0 & 0 & s-1 & -1 \\ 0 & 0 & 0 & s-1 \end{vmatrix} = s^4 - 2s^3 + s^2 + 0s$$

From the classical Cayley-Hamilton theorem, we have $\boldsymbol{A}^4 - 2\boldsymbol{A}^3 + \boldsymbol{A}^2 = \boldsymbol{0}$. By applying Lemma 4, we obtain $(\boldsymbol{A}^{\oplus})^3 - 2(\boldsymbol{A}^{\oplus})^2 + \boldsymbol{A}^{\oplus} = \boldsymbol{0}$.

Note that, if the characteristic polynomials of $\boldsymbol{A}^{\oplus}$ and $\boldsymbol{A}^{\mathrm{D}}$ is

$$p_{\boldsymbol{A}^{\oplus}}(s) = \det(s\boldsymbol{E}_n - \boldsymbol{A}^{\oplus}) = s^n + b_{n-1}s_{n-1} + \ldots + b_1 s \tag{12}$$

$$p_{\boldsymbol{A}^{\mathrm{D}}}(s) = \det(s\boldsymbol{E}_n - \boldsymbol{A}^{\mathrm{D}}) = s^n + c_{n-1}s_{n-1} + \ldots + c_1 s \tag{13}$$

respectively, we cannot obtain

$$p_{\boldsymbol{A}^{\oplus}}(s) = b_1\boldsymbol{A}^n + b_2\boldsymbol{A}^{n-1} + \ldots + b_{n-1}\boldsymbol{A}^2 + \boldsymbol{A} \tag{14}$$

$$p_{\boldsymbol{A}^{\mathrm{D}}}(s) = c_1\boldsymbol{A}^n + c_2\boldsymbol{A}^{n-1} + \ldots + c_{n-1}\boldsymbol{A}^2 + \boldsymbol{A} \tag{15}$$

Example 2 Let

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

It is easy to confirm that the core-EP inverse $\boldsymbol{A}^{\oplus}$ is

$$\boldsymbol{A}^{\oplus} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \boldsymbol{A}^{\mathrm{D}}$$

Then

$$p_{\boldsymbol{A}^{\mathrm{D}}}(s) = p_{\boldsymbol{A}^{\oplus}}(s) = \begin{vmatrix} s & 0 \\ 0 & s \end{vmatrix} = s^2 + 0s$$

but

$$f_{A^{D}}(s)=f_{A^{\oplus}}(s)=0A^2+A=A\neq 0$$

Theorem 3 Let $A\in C_{n,n}$ and $\mathrm{Ind}(A)=k$. Then the characteristic polynomial of $A^{\oplus}\in C_{n,n}$ is

$$p_{A^{\oplus}}(s)=s^n+b_{n-1}s_{n-1}+\ldots+b_{n-\mathrm{rk}(A^k)}s^{n-\mathrm{rk}(A^k)} \quad (16)$$

Furthermore,

$$b_{n-\mathrm{rk}(A^k)}A^n+\ldots+b_{n-1}A^{n-\mathrm{rk}(A^k)+1}+A^{n-\mathrm{rk}(A^k)}=0 \quad (17)$$

Proof Let the core-EP decomposition of A, i. e. $A=A_1+A_2$, be as in Lemma 2. Then

$$p_{A^{\oplus}}(s)=\det\left(sE_n-U\begin{bmatrix}T^{-1} & 0\\ 0 & 0\end{bmatrix}U^*\right)=s^{n-\mathrm{rk}(A^k)}\det(sE_{\mathrm{rk}(A^k)}-T^{-1})$$

Therefore, we obtain (16). Using (16) and Theorem 1, we obtain

$$(A^{\oplus})^n+b_{n-1}(A^{\oplus})^{n-1}+\ldots+b_{n-\mathrm{rk}(A^k)}(A^{\oplus})^{n-\mathrm{rk}(A^k)}=0 \quad (18)$$

that is,

$$U\begin{bmatrix}T^{-n} & 0\\ 0 & 0\end{bmatrix}U^*+b_{n-1}U\begin{bmatrix}T^{-(n-1)} & 0\\ 0 & 0\end{bmatrix}U^*+\ldots+b_{n-\mathrm{rk}(A^k)}U\begin{bmatrix}T^{-(n-\mathrm{rk}(A^k))} & 0\\ 0 & 0\end{bmatrix}U^*=0 \quad (19)$$

Post-multiplying (19) by

$$A^{n+n-\mathrm{rk}(A^k)}=U\begin{bmatrix}T^{n+n-\mathrm{rk}(A^k)} & \Phi_{n+n-\mathrm{rk}(A^k)}\\ 0 & 0\end{bmatrix}U^*=0$$

we have (17).

Theorem 4 Let $A\in C_{n,n}$, $\mathrm{Ind}(A)\leqslant 1$ and the characteristic polynomial of $A^{\oplus}\in C_{n,n}$ be as in (12). Then

$$f_{A^{\oplus}}(A)=b_1A^n+b_2A^{n-1}+\ldots+b_{n-1}A^2+A=0 \quad (20)$$

Theorem 5 Let $A\in C_{n,n}$ and $\mathrm{Ind}(A)=k$. Then the characteristic polynomial of $A^{D}\in C_{n,n}$ is

$$p_{A^{D}}(s)=\det(sE_n-A^{D})=s^n+c_{n-1}s_{n-1}+\ldots+c_{n-\mathrm{rk}(A^k)}s^{n-\mathrm{rk}(A^k)} \quad (21)$$

Furthermore,

$$c_{n-\mathrm{rk}(A^k)}A^n+\ldots+c_{n-1}A^{n-\mathrm{rk}(A^k)+1}+A^{n-\mathrm{rk}(A^k)}=0 \quad (22)$$

Proof Let the core-nilpotent decomposition of A, $A=\hat{A}_1+\hat{A}_2$ be as in Lemma 1. Then

$$p_{A^{D}}(s)=\det\left(sE_n-P\begin{bmatrix}\Sigma^{-1} & 0\\ 0 & 0\end{bmatrix}P^{-1}\right)=\det\left(\begin{bmatrix}sE_{\mathrm{rk}(A^k)}-\Sigma^{-1} & 0\\ 0 & E_{n-\mathrm{rk}(A^k)}\end{bmatrix}\right)=s^{n-\mathrm{rk}(A^k)}\det(sE_{\mathrm{rk}(A^k)}-\Sigma^{-1})$$

Therefore, we obtain (21). Using (21) and Theorem 1 we obtain

$$(A^{D})^n+b_{n-1}(A^{D})^{n-1}+\ldots+b_{n-\mathrm{rk}(A^k)}(A^{D})^{n-\mathrm{rk}(A^k)}=0$$

that is,

$$P\begin{bmatrix}\Sigma^{-n} & 0\\ 0 & 0\end{bmatrix}P^{-1}+b_{n-1}P\begin{bmatrix}\Sigma^{-(n-1)} & 0\\ 0 & 0\end{bmatrix}P^{-1}+\ldots+b_{n-\mathrm{rk}(A^k)}P\begin{bmatrix}\Sigma^{-(n-\mathrm{rk}(A^k))} & 0\\ 0 & 0\end{bmatrix}P^{-1}=0$$

Post-multiplying the above equation by

$$A^{n+n-\mathrm{rk}(A^k)}=P\begin{bmatrix}\Sigma^{n+n-\mathrm{rk}(A^k)} & 0\\ 0 & 0\end{bmatrix}P^{-1}=0$$

Therefore, we obtain (22).

Theorem 6 Let $A\in C_{n,n}$ and $\mathrm{Ind}(A)\leqslant 1$ and the characteristic polynomial of $A^{D}\in C_{n,n}$ be as in (13). Then

$$f_{A^{D}}(A)=c_1A^n+c_2A^{n-1}+\ldots+c_{n-1}A^2+A=0 \quad (23)$$

Let $A\in C_{n,n}$ and $\mathrm{Ind}(A)=k$. Then the DMP inverse of A is $A^{d,+}=A^{D}AA^{+}$[3]. Since $(A^{d,+})^2=A^{D}AA^{+}A^{D}AA^{+}=A^{D}AA^{+}AA^{D}A^{+}=(A^{D})^2AA^{+}$, we obtain

$$(A^{d,+})^p=(A^{D})^pAA^{+} \quad (24)$$

where p is a positive integer.

Theorem 7 Let the characteristic polynomial of $A\in C_{n,n}$ be as in (1). Then

$$f_A(A^{d,+})=a_1(A^{d,+})^n+a_2(A^{d,+})^{n-1}+\ldots+a_{n-1}(A^{d,+})^2+A^{d,+}=0 \quad (25)$$

Proof By applying (1) and Theorem 2, we obtain

$$a_1(A^{D})^nAA^{+}+a_2(A^{D})^{n-1}AA^{+}+\ldots+a_{n-1}(A^{D})^2AA^{+}+A^{D}AA^{+}=0$$

From (24), we can obtain (25).

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矩阵 core-EP 逆和 DMP 逆的广义 Cayley-Hamilton 定理

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摘要: 利用经典的 Cayley-Hamilton 定理，给出了矩阵 core-EP 逆和 DMP 逆的多项式方程. 设奇异矩阵 $\boldsymbol{A}$ 的特征多项式为 $p_{A}(s)=\det(s\boldsymbol{E}_n-\boldsymbol{A})=s^n+a_{n-1}s_{n-1}+\cdots+a_1 s$，则 $f_A(\boldsymbol{A}^{\oplus})=\boldsymbol{0}$ 和 $f_A(\boldsymbol{A}^{\mathrm{d},+})=\boldsymbol{0}$，其中 $f_A(\boldsymbol{A})=a_1x^n+a_2x^{n-1}+\cdots+a_{n-1}x^2+x$，$\boldsymbol{A}^{\oplus}$ 和 $\boldsymbol{A}^{\mathrm{d},+}$ 分别是 $\boldsymbol{A}$ 的 core-EP 逆和 DMP 逆. 并进一步讨论了 $\boldsymbol{A}^{\mathrm{D}}\in\boldsymbol{C}_{n,n}$ 和 $\boldsymbol{A}^{\oplus}\in\boldsymbol{C}_{n,n}$ 的特征多项式的性质.

关键词: Cayley-Hamilton 定理；特征方程；Drazin 逆；DMP 逆；core-EP 逆
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