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Linear Weingarten spacelike hypersurface in locally symmetric Lorentz space

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Abstract: The rigidity of spacelike hypersurface M^n immersed in locally symmetric space M_1^{n+1} is investigated, where the (normalized) scalar curvature R and mean curvature H of M^n satisfy $R = aH + b$, and a, b are real constants. First, an estimate of the upper bound of the function $L(nH)$ is given, where L is a second-order differential operator. Then, under the assumption that the square norm of the second fundamental form is bounded by a given positive constant, it is proved that M^n must be either totally umbilical or contain two distinct principle curvatures, one of which is simple. Moreover, a similar result is obtained for complete noncompact spacelike hypersurfaces in locally symmetric Einstein spacetime. Hence, some known rigidity results for hypersurface with constant scalar curvature are extended for the linear Weingarten case.

Key words: spacelike hypersurface; linear Weingarten; locally symmetric Lorentz space

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Let M_1^{n+1} be an $(n+1)$-dimensional pseudo-Riemannian manifold of index 1, i. e. Lorentz space. When the Lorentz space M_1^{n+1} has constant curvature c, we call it a Lorentz space form, denoted by $M_1^{n+1}(c)$, with de Sitter space $S_1^{n+1}(1)$ and anti-de Sitter space $H_1^{n+1}(-1)$.

Suppose that M^n is a spacelike hypersurface immersed in M_1^{n+1}, where M^n is said to be spacelike if the metric on M^n induced from that on M_1^{n+1} is positive definite. The spacelike hypersurface with constant scalar curvature or constant mean curvature has been extensively studied in de Sitter space $S_1^{n+1}(1)$ [1], anti-de Sitter space $H_1^{n+1}(-1)$ [2] and the general Lorentz space[3]. It is worth noting that all the above results were obtained for the case where the ambient manifolds possess very good symmetric properties. Furthermore, when M_1^{n+1} is locally symmetric but does not have symmetry in general, many results can be found. For example, Liu et al. [4−5] obtained some rigidity theorems independently for complete noncompact M^n with a constant scalar curvature, where M_1^{n+1} satisfies the following two conditions: Condition 1) For any spacelike vector μ and any time-like vector ν, $\overline{K}(\mu, \nu) = -c_1/n$; Condition 2) For any space-like vectors μ and ν, $\overline{K}(\mu, \nu) \geqslant c_2$, where c_1, c_2 are real constants; and $\overline{K}$ is the sectional curvature of M_1^{n+1}.

On the other hand, as a natural generalization of hypersurface with constant scalar curvature or with constant mean curvature, the linear Weingarten hypersurface has been extensively studied during the past decades[6−10]. A hypersurface is said to be linear Weingarten if its (normalized) scalar curvature R and its mean curvature H satisfy $R = aH + b$, where a and b are real constants. Motivated by this observation, Yang[10] extended the theorems in Refs. [4 −5] to the linear Weingarten case.

Recently, Wang and Liu[11] investigated the rigidity problems for compact M^n with constant scalar curvature in M_1^{n+1} which satisfies Condition 1) and Condition 3): For any spacelike vectors μ and ν, $\overline{K}(\mu, \nu) \leqslant c_2$.

Inspired by the above observations, we continue to study the rigidity for linear Weingarten spacelike hypersurface M^n in M_1^{n+1}, where M_1^{n+1} is locally symmetric and satisfies Condition 1) and Condition 3).

1 Preliminaries

Let M_1^{n+1} be a locally symmetric space and M^n be an n-dimensional spacelike hypersurface immersed in M_1^{n+1}. For any $p \in M^n$, we choose a local orthonormal frame $e_1, e_2, \ldots, e_{n+1}$ in M^n around p such that $e_1, e_2, \ldots, e_n$ are tangent to M^n. Let $\omega_1, \omega_2, \ldots, \omega_{n+1}$ be the corresponding dual co-frame. We shall use the following standard convention for indices:

$$1 \leqslant A, B, C, \ldots \leqslant n+1, \qquad 1 \leqslant i, j, k, \ldots \leqslant n$$

The structure equations of M_1^{n+1} are given by

$$\mathrm{d}\omega_A = -\sum_B \omega_{AB} \wedge \omega_B, \quad \omega_{AB} + \omega_{BA} = 0$$

$$\mathrm{d}\omega_{AB} = -\sum_C \omega_{AC} \wedge \omega_{CB} - \frac{1}{2}\sum_{C,D} \varepsilon_C \varepsilon_D \overline{K}_{ABCD} \omega_C \wedge \omega_D$$

where $\overline{K}_{ABCD}$ are the components of the curvature tensor of M_1^{n+1}.

Restricting these forms to M^n, we have $\omega_{n+1} = 0$. Since

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$0 = d\omega_{n+1} = -\sum_i \omega_{n+1i} \wedge \omega_i$, by using Cartan's lemma, we have

$$\omega_{n+1i} = \sum_j h_{ij}\omega_j \quad h_{ij} = h_{ji}$$

Let $A = \sum_{i,j} h_{ij}\omega_i\omega_j e_{n+1}$, $\boldsymbol{H} = \frac{1}{n}\sum_i h_{ii}e_{n+1}$, $H = |\boldsymbol{H}| = \frac{1}{n}\sum_i h_{ii}$ be the second fundamental form, the mean curvature vector and the mean curvature of M^n, respectively.

The structure equations of M^n are

$$d\omega_i = -\sum_{j=1}^n \omega_{ij} \wedge \omega_j, \quad \omega_{ij} + \omega_{ji} = 0$$

$$d\omega_{ij} = -\sum_{k=1}^n \omega_{ik} \wedge \omega_{kj} + \frac{1}{2}\sum_{k,l=1}^n R_{ijkl}\omega_k \wedge \omega_l$$

The Gauss equations are

$$R_{ijkl} = \overline{K}_{ijkl} + (h_{ik}h_{jl} - h_{il}h_{jk})$$

$$n(n-1)R = \sum_{i,j} \overline{K}_{ijij} - n^2H^2 + S \qquad (1)$$

where R is the normalized scalar curvature and $S = \sum_{i,j} h_{ij}^2$.

The Codazzi and Ricci equations are

$$h_{ijk} - h_{ikj} = \overline{K}_{(n+1)ijk}$$

$$\overline{K}_{(n+1)ijk;l} = \overline{K}_{(n+1)ijkl} + \overline{K}_{(n+1)i(n+1)k}h_{jl} + \overline{K}_{(n+1)ij(n+1)}h_{kl} - \sum_m \overline{K}_{mijk}h_{ml}$$

where the covariant derivative of h_{ij} is defined as

$$\sum_k h_{ijk}\omega_k = dh_{ij} - \sum_k h_{kj}\omega_{ki} - \sum_k h_{ik}\omega_{kj}$$

Similarly, the components h_{ijkl} of the second derivative $\nabla^2 h$ are given as

$$\sum_l h_{ijkl}\omega_l = dh_{ijk} - \sum_l h_{ljk}\omega_{li} - \sum_l h_{ilk}\omega_{lj} - \sum_l h_{ijl}\omega_{lk}$$

The Laplacian Δh_{ij} of h_{ij} is defined as $\Delta h_{ij} = \sum_k h_{ijkk}$. By direct calculation, we obtain

$$\Delta h_{ij} = \sum_k [(h_{ijkk} - h_{ikjk}) + (h_{ikjk} - h_{ikkj}) + (h_{ikkj} - h_{kkij}) + h_{kkij}] = (nH)_{ij} + nH\overline{K}_{n+1in+1j} - \sum_k h_{ij}\overline{K}_{n+1kn+1k} + nH\sum_k h_{ik}h_{kj} - Sh_{ij} + \sum_k (h_{mi}R_{mkjk} + h_{mj}R_{mkik} + 2h_{km}R_{mijk})$$

We then choose a local frame of orthonormal vector fields $\{e_i\}$ such that at $p \in M^n$,

$$h_{ij} = \lambda_i\delta_{ij}$$

Then it follows, at p, that

$$\frac{1}{2}\Delta S = \frac{1}{2}\sum_{i,j}\Delta h_{ij}^2 = \sum_{i,j,k} h_{ijk}^2 + \sum_{i,j} h_{ij}\Delta h_{ij} = \sum_{i,j,k} h_{ijk}^2 + \sum_i \lambda_i(nH)_{ii} - S^2 + nH\sum_i \lambda_i^3 + nH\sum_i \lambda_i\overline{K}_{n+1in+1i} - S\sum_i \overline{K}_{n+1in+1i} + \sum_{i,j} (\lambda_i - \lambda_j)^2\overline{K}_{ijij} \qquad (2)$$

Set $\varphi_{ij} = h_{ij} - H\delta_{ij}$, and it is easy to confirm that $\boldsymbol{\varphi}$ is traceless and

$$|\boldsymbol{\varphi}|^2 = \sum_{i,j} (\varphi_{ij})^2 = S - nH^2$$

where $\boldsymbol{\varphi} = (\varphi_{ij})$ is a real matrix. Moreover, $|\boldsymbol{\varphi}|^2 = S - nH^2 \geqslant 0$ with equality holds if and only if M^n is totally umbilical.

Following Cheng and Yau[12], we introduce the operator W associated with $\boldsymbol{\varphi}$ acting on any smooth function f by

$$W(f) = \sum_{i,j} (nH\delta_{ij} - h_{ij})f_{ij} \qquad (3)$$

Then, setting $f = nH$ in Eq. (3), we obtain

$$W(nH) = \sum_{i,j} (nH\delta_{ij} - h_{ij})(nH)_{ij} = \frac{1}{2}\Delta(nH)^2 - \sum_i (nH)_i^2 - \sum_i \lambda_i(nH)_{ii} \qquad (4)$$

Lemma 1[13] Let $\beta_1, \beta_2, \ldots, \beta_n$ be real numbers such that $\sum_{i=1}^n \beta_i = 0$. Then

$$-\frac{n-2}{\sqrt{n(n-1)}}\left(\sum_{i=1}^n \beta_i^2\right)^{3/2} \leqslant \sum_{i=1}^n \beta_i^3 \leqslant \frac{n-2}{\sqrt{n(n-1)}}\left(\sum_{i=1}^n \beta_i^2\right)^{3/2}$$

Moreover, the equality holds if and only if at least $(n-1)$ of β_i's are equal.

Lemma 2[11] Let $M^n (n>2)$ be a spacelike hypersurface immersed in locally symmetric Lorentz space M_1^{n+1}. If $h_{ijk} \geqslant 0$, then $\sum_{i,j,k} h_{ijk}^2 \leqslant n^2|\nabla H|^2$.

Lemma 3[6] Let $\boldsymbol{X}$ be a smooth vector field on the complete non-compact Riemannian manifold M^n, such that $\mathrm{div}_M\boldsymbol{X}$ does not change sign on M^n, where div represents the divergence operator. If $|\boldsymbol{X}| \in \mathscr{L}^1(M)$, then $\mathrm{div}_M\boldsymbol{X} = 0$.

Lemma 4 Let $M^n (n>2)$ be a spacelike hypersurface immersed in locally symmetric Lorentz space M_1^{n+1} which satisfies Condition 1) and Condition 3). Then

$$\frac{1}{2}\Delta S \leqslant \sum_{i,j,k} h_{ijk}^2 + \sum_i \lambda_i(nH)_{ii} + nc(S - nH^2) + (S^2 - nH\sum_i \lambda_i^3)$$

where $c = 2c_2 + c_1/n$.

Proof First, we observe that the local symmetry of M_1^{n+1} implies that $\overline{K}_{ABCD;E} = 0$, thus

$$\sum_{i,k} \lambda_i(\overline{K}_{(n+1)iik;k} + \overline{K}_{(n+1)kik;i}) = 0 \qquad (5)$$

Since M_1^{n+1} satisfies Condition 1) and Condition 3),

$$nH\sum_i \lambda_i\overline{K}_{(n+1)ii(n+1)} + S\sum_i \overline{K}_{(n+1)i(n+1)i} = -c_1(S - nH^2) \qquad (6)$$

$$-2\sum_{j,k}(\lambda_j\lambda_k-\lambda_k^2)\bar{K}_{kjjk}\leqslant c_2\sum_{j,k}(\lambda_j-\lambda_k)^2=2nc_2(S-nH^2)\tag{7}$$

Then the lemma can be proven easily by substituting Eqs. (5) to (7) into Eq. (2).

Lemma 5[14] Let M^n be a linear Weingarten spacelike hypersurface immersed in locally symmetric Lorentz space M_1^{n+1} with $b<\frac{1}{n(n-1)}\sum_{i,j}\bar{K}_{ijij}$. Then $L=W+((n-1)a\Delta/2)$ is elliptic.

Lemma 6 Let $M^n(n>2)$ be a spacelike hypersurface immersed in locally symmetric Lorentz space M_1^{n+1} which satisfies Condition 1) and Condition 3) with $h_{ijk}\geqslant 0$ and $R=aH+b$. Then

$$L(nH)\leqslant|\varphi|^2\left(nc-nH^2+|\varphi|^2+\frac{n(n-2)}{\sqrt{n(n-1)}}|H||\varphi|\right)\tag{8}$$

where $c=2c_2+c_1/n$.

Proof First, we obtain from Eq. (1) that

$$n^2H^2=\sum_{i,j}\bar{K}_{ijij}-n(n-1)R+S=\sum_{i,j}\bar{K}_{ijij}+S-n(n-1)(aH+b)\tag{9}$$

Since M_1^{n+1} is locally symmetric, $\sum_{i,j}\bar{K}_{ijij}$ is a constant[10]. Substituting Eq. (9) into (4), we have

$$L(nH)=\frac{1}{2}\Delta S-n^2|\nabla H|^2-\sum_i\lambda_i(nH)_{ii}\tag{10}$$

Applying Lemmas 2 and 4 to Eq. (10), we obtain

$$L(nH)\leqslant nc(S-nH^2)+(S^2-nH\sum_i\lambda_i^3)\tag{11}$$

Let $\mu_i=\lambda_i-H$, we can obtain

$$\sum_i\mu_i=0,\quad |\varphi|^2=\sum_i\mu_i^2$$
$$\sum_i\lambda_i^3=\sum_i\mu_i^3+3H|\varphi|^2+nH^3$$

Using Lemma 1, we obtain that

$$-nH\sum_i\lambda_i^3=-n^2H^4-3nH^2|\varphi|^2-nH\sum_i\mu_i^3\leqslant -n^2H^4-3nH^2|\varphi|^2+\frac{n(n-2)}{\sqrt{n(n-1)}}|H||\varphi|^3\tag{12}$$

Substituting (12) into (11), we have

$$L(nH)\leqslant nc|\varphi|^2+(|\varphi|^2+nH^2)2-n^2H^4-3nH^2|\varphi|^2+\frac{n(n-2)}{\sqrt{n(n-1)}}|H||\varphi|^3=|\varphi|^2\left(nc-nH^2+|\varphi|^2+\frac{n(n-2)}{\sqrt{n(n-1)}}|H||\varphi|\right)$$

2 Main Results

Theorem 1 Let M_1^{n+1} be a locally symmetric Lorentz space which satisfies Condition 1) and Condition 3). Suppose that $M^n(n>2)$ is a compact linear Weingarten spacelike hypersurface immersed in M_1^{n+1} with $h_{ijk}\geqslant 0$. If $S\leqslant -2\sqrt{n-1}c$, $c=2c_2+\frac{c_1}{n}\leqslant 0$, then either $S=nH^2$ or $S=-2\sqrt{n-1}c$. When $S=nH^2$, M^n is totally umbilical. When $S=-2\sqrt{n-1}c$, M^n has two distinct principle curvatures, one of which is simple.

Proof First, we consider the quadratic form

$$Q(x,y)=-x^2+\frac{n-2}{\sqrt{n-1}}xy+y^2$$

By using the orthogonal transformation

$$\left.\begin{aligned}u&=\frac{1}{\sqrt{2n}}\{(1+\sqrt{n-1})y-(1-\sqrt{n-1})x\}\\ v&=\frac{1}{\sqrt{2n}}\{(\sqrt{n-1}-1)y-(1+\sqrt{n-1})x\}\end{aligned}\right\}$$

We obtain that

$$Q(x,y)=\frac{n}{2\sqrt{n-1}}(u^2-v^2)$$

Set $x=\sqrt{nH^2}$, $y=|\varphi|$. It is not difficult to verify that $u^2+v^2=x^2+y^2=|\varphi|^2+nH^2=S$. Then

$$nc-nH^2+|\varphi|^2+\frac{n(n-2)}{\sqrt{n(n-1)}}|H||\varphi|=nc+\frac{n}{2\sqrt{n-1}}(u^2-v^2)\leqslant nc+\frac{n}{2\sqrt{n-1}}(u^2+v^2)=nc+\frac{n}{2\sqrt{n-1}}S\tag{13}$$

Substituting(13) into (8), we obtain

$$L(nH)\leqslant|\varphi|^2\left(nc+\frac{n}{2\sqrt{n-1}}S\right)\tag{14}$$

On the other hand, since L is self-adjoint and M^n is compact,

$$\int_{M^n}L(nH)=0\tag{15}$$

Then, from (14), (15) and $S\leqslant -2\sqrt{n-1}c$, we obtain that

$$|\varphi|^2\left(nc+\frac{n}{2\sqrt{n-1}}S\right)\equiv 0$$

If $S<-2\sqrt{n-1}c$, then $|\varphi|^2\equiv 0$ and M^n is totally umbilical. If $S=-2\sqrt{n-1}c$, then all the above inequalities become equalities. Especially, when equality in Lemma 1 holds, we then obtain that M^n has two distinct principle curvatures, one of which is simple.

Furthermore, when M^n is complete noncompact, we have the following extension of Theorem 1.

Theorem 2 Let M_1^{n+1} be a locally symmetric Lorentz space which satisfies Condition 1) and Condition 3).

Suppose that $M^n(n>2)$ is a complete noncompact linear Weingarten spacelike hypersurface immersed in M_1^{n+1} with $h_{ijk}\geqslant 0$. If H can attain the maximum on M^n and $S\leqslant -2\sqrt{n-1}c$, $c=2c_2+\frac{c_1}{n}\leqslant 0$, then either $S=nH^2$ or $S=-2\sqrt{n-1}c$. When $S=nH^2$, M^n is totally umbilical. When $S=-2\sqrt{n-1}c$, M^n has two distinct principle curvatures, one of which is simple.

Proof According to the proof in Theorem 1, we obtain

$$L(nH)\leqslant|\varphi|^2\left(nc+\frac{n}{2\sqrt{n-1}}S\right)$$

Due to the fact that $S\leqslant -2\sqrt{n-1}c$, we can immediately conclude that

$$L(nH)\leqslant\frac{n}{2\sqrt{n-1}}|\varphi|^2(2\sqrt{n-1}c+S)\leqslant 0 \quad (16)$$

Noting that L is elliptic and H attains its maximum on M^n, by using the maximum principle, we can obtain that H is a constant. Consequently,

$$\sum_{i,j,k}h_{ijk}^2=n^2|\nabla H|^2=0$$

Hence, λ_i is constant for each $i=1, 2, \ldots, n$. Furthermore, $L(nH)=0$ and we obtain from (16) that

$$|\varphi|^2(2\sqrt{n-1}c+S)=0 \quad (17)$$

By using the same argument as in Theorem 1, the proof can be completed easily.

Remark 1 Since S is bounded, H is bounded as well. Then H can attain its maximum on M^n since $h_{ijk}\geqslant 0$. Hence, the assumption that H can attain the maximum on M^n in Theorem 2 can be removed. So, the main difference between Theorem 2 and Theorem 1.6 in Ref. [10] lies in the assumption that sup H is attained at some points or not.

If the metric and Ricci tensors of a Lorentz space are homotetic[15], we call it Einstein spacetime. For the spacelike hypersurface in Einstein spacetime, we have the following result.

Theorem 3 Let M_1^{n+1} be a locally symmetric Einstein spacetime which satisfies Condition 1) and Condition 3). Suppose that $M^n(n>2)$ is a complete non-compact linear Weingarten spacelike hypersurface immersed in M_1^{n+1} with $|\nabla H|\in\mathscr{L}^1(M)$, where $\mathscr{L}^1(M)$ represents the space of Lebesgue integrable functions on M^n. If $S\leqslant -2\sqrt{n-1}c$, $c=2c_2+\frac{c_1}{n}\leqslant 0$, then either $S=nH^2$ or $S=-2\sqrt{n-1}c$. When $S=nH^2$, M^n is totally umbilical. When $S=-2\sqrt{n-1}c$, M^n has two distinct principle curvatures, one of which is simple.

Proof According to Ref. [15], we have

$$L(nH)=\mathrm{div}_M(P(\nabla H))$$

where $P=\left(n^2H+\frac{n(n-1)}{2}a\right)I-nA$ and I denotes the identity operator. Furthermore, since $S\leqslant -2\sqrt{n-1}c$ and $nH^2\leqslant S$, both H and A are bounded on M^n. Hence, the operator P is bounded. Then, from $|\nabla H|\in\mathscr{L}^1(M)$, we obtain that

$$|P(\nabla H)|\in\mathscr{L}^1(M) \quad (18)$$

Thus, from (16), (18) and Lemma 3, we obtain that $L(nH)=0$. By using the same argument as in Theorem 2, the proof is completed easily.

Remark 2 The Lorentz space form M_1^{n+1} satisfies both Condition 2) and Condition 3), where $-c_1/n=c_2=c$.

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局部对称 Lorentz 空间中线性 Weingarten 类空超曲面

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摘要: 研究了局部对称 Lorentz 空间 M_1^{n+1} 中类空超曲面 M^n 的刚性问题,其中 M^n 的数量曲率 R 和平均曲率 H 满足线性关系 $R = aH + b$, a, b 是实常数. 首先,给出函数 $L(nH)$ 上界的估计值,其中 L 是二阶微分算子. 若 M^n 第二基本形式的平方范数小于或等于一个给定的正常数,证明了: M^n 一定是全脐地,或者含有 2 个不同的主曲率,且其中一个主曲率是单的. 此外,还得到了关于局部对称爱因斯坦时空中完备非紧类空超曲面类似的结果. 因此,具有常数量曲率超曲面的刚性结果被推广到线性 Weingarten 情形.

关键词: 类空超曲面; 线性 Weingarten; 局部对称 Lorentz 空间

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