

Availability-based maintenance optimization under unequal inspection period and imperfect repair

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Abstract: Taking the multi-component system as research object, a maintenance optimization model based on the unequal inspection period and imperfect repair is established by considering the requirement of expected availability for improving the system's availability. An age reduction factor is used to describe the effect of imperfect repair, and the modelling approach for the unequal inspection period is proposed. Unavailable situations are classified into three kinds of independent cases, and the availability is calculated accordingly. Based on the analysis of the relationship between the unavailable cases and the unequal inspection period, an optimization model under imperfect repair is established to optimize the system's expected availability. A case study of a wind turbine is provided, and three key components, i. e. gearbox, generator and spindle, are considered. The optimization results of the unequal inspection period model and the equal inspection period model are compared. The results show that the unequal inspection period model based on availability can update the maintenance plan so as to optimize maintenance activities and improve the system's availability.

Key words: age reduction; inspection period; unequal period maintenance; maintenance optimization; availability

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Most engineering equipment are multi-component products, such as CNC machine tools, wind turbines, construction machinery, radars, etc. For these types of equipment, the purchasing price, operation and maintenance (O & M) costs are usually high. Considering that a component's performance will degrade gradually until a failure occurs, it is necessary to carry out reasonable maintenance to restore the system's function. Scientific maintenance policy is crucial for maintaining the availability and reducing the O & M cost^[1-2]. Besides, maintenance is also helpful to maintain and improve the availability^[3].

Some studies on maintenance optimization by consider-

ing the availability's constraint have been done. Maciejewski et al.^[4] proposed an approach to estimate the system's availability, and the Monte Carlo simulation and statistical analysis were used to verify the method. Do et al.^[5] proposed a group maintenance plan for multi-component systems, where the constraint of availability was considered. Kumar et al.^[6] analyzed availability via a semi-Markov process, and the stable solution of the system's availability was obtained. In Ref. [7], a system availability model was established by using a Markov process and universal generating function, and the total maintenance cost was selected as the constraint.

Meanwhile, Moura et al.^[8] adopted a semi-Markov process to evaluate the system's availability. Based on the mixed risk rate model, Khatab et al.^[9] studied imperfect preventive maintenance (PM) for mechanical systems, and an availability model was established. Wang et al.^[10] established a long-term availability model, and numerical simulations were used to optimize availability under imperfect repair. Lü et al.^[11] proposed an inspection policy, and a maintenance optimization model was developed. Based on the Petri nets theory, an availability model was proposed to calculate availability under different maintenance strategies in Ref. [12]. With the aim to maximize the system's availability, Ali et al.^[13] proposed a selective maintenance policy to determine the number of components to be replaced or repaired within the limited time interval. In Ref. [14], the availability and maintenance policies undergoing periodic inspections were studied for a competing-risk system. Aiming to improve the availability of a distribution system, Tiwary et al.^[15] proposed an approach to optimize the inspection and repair.

In most of the above studies, the inspection and maintenance period were supposed to be equal. This is impractical because their aging speed will be accelerated with the increase of products' usage. Thus, more frequent inspection and maintenance are needed. Up to now, studies on maintenance optimization for multi-component systems considering the unequal inspection period and availability requirements are quite limited.

In this study, an age reduction factor is introduced to describe the effect of imperfect repair, and a maintenance optimization model based on the unequal inspection period is proposed, in order to develop reasonable maintenance decision and achieve the expected system's availability.

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1 Modeling for Maintenance Optimization

1.1 Inspection policy and assumptions

For a multi-component product, it usually needs to be checked to ensure its availability. Generally, the inspection obeys the following assumptions:

- 1) The failure can be detected at time $t_i (i = 1, 2, \dots, N)$, and the inspection period T_i is varied;
- 2) The inspection is perfect, and the possible failure can be found at the end of the inspection;
- 3) If a failure is found during the inspection, the component will be maintained just after the inspection, and repair time γ is a random variable;
- 4) The inspection time θ of each inspection is a constant;
- 5) Before the inspection, the system may be available or unavailable, while its state does not change during the inspection.

1.2 Age reduction

By adopting imperfect repair, the performance of a component or system can be restored to some extent. In order to describe the change of the failure rate after repair, an age reduction factor is introduced. The value of the age reduction factor is set to be $[0, 1]$, where “0” means that the system is restored to the state of as bad as old (ABAO) and “1” means that the system is restored to the state of as good as new (AGAN). Usually, the system will be restored to somewhere between ABAO and AGAN, and the value of the age reduction factor depends on the effect of imperfect repair, as shown in Fig. 1.

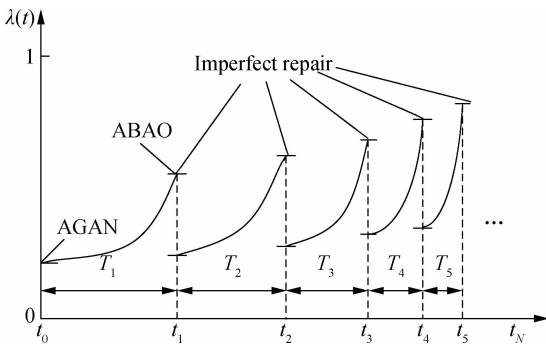


Fig. 1 Failure rate and maintenance

From Fig. 1, with the continuous use of a product, in order to ensure that it has high availability, the inspection period should be continuously shortened, i. e., $T_1 > T_2 > \dots > T_N$. It is assumed that there is a proportional relationship between the adjacent inspection periods, i. e., $T_2 = \alpha_1 T_1$, $T_3 = \alpha_2 T_2$, ..., $T_N = \alpha_{N-1} T_{N-1}$, the inspection period T_i can be obtained as

$$T_i = \alpha_1 \alpha_2 \dots \alpha_{i-1} T_1 \quad i = 1, 2, \dots, N \quad (1)$$

where $T_i = t_i - t_{i-1}$ is the i -th inspection period, which includes working time and nonworking time; t_i is the time

point corresponding to the i -th inspection; α_i is the age reduction factor during the i -th inspection period; N is the total number of inspection periods.

1.3 Model of system availability

According to the above assumptions, the availability is a periodic function of T_i . By adding the effective working time during each inspection period, we can obtain the total effective working time. Thus, the system's availability at any time $x (x = t - t_{i-1}, 0 \leq x \leq T_i, t \geq t_{i-1})$ between two inspections can be calculated based on Fig. 2.

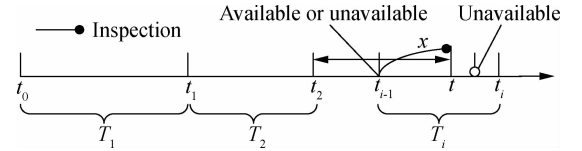


Fig. 2 The inspection process of system

At operation time x which is started from the last inspection point, the system may be down due to the following reasons:

Case 1 The system is in repair. If a failure is found during the inspection and it will be maintained immediately after the inspection, the repair time is γ and the inspection time is θ . If $\gamma > x - \theta$, the system is unavailable. Thus, Case 1 = (failure is found during the inspection) \cap ($\gamma > x - \theta$), as shown in Fig. 3(a).

Case 2 The system has failed, while the failure has not been found. It can be further divided into the following two subcases:

Case 2.1 Case 2.1 = (no failure is found during the inspection) \cap (a failure occurs during the $x - \theta$ period after the inspection), as shown in Fig. 3(b).

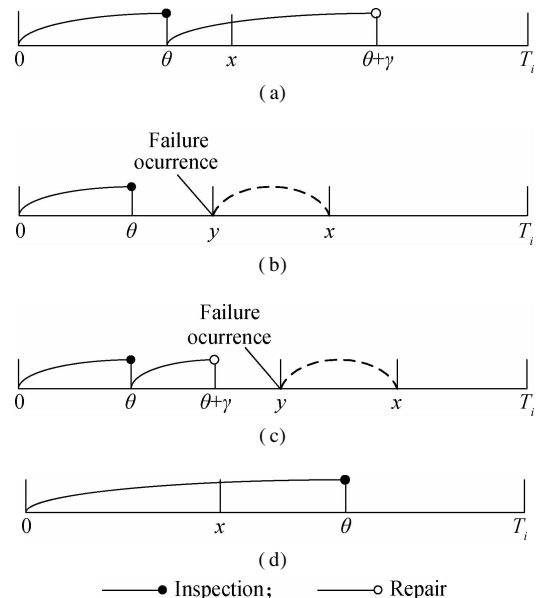


Fig. 3 Three independent cases for the unavailable situations. (a) Case 1; (b) Case 2.1; (c) Case 2.2; (d) Case 3

Case 2.2 The failure is found during the inspection and repaired immediately after the inspection, and the re-

pair is completed at time y ($\theta \leq y \leq x$). After this, the system fails again during the $x - y$ period. Thus, Case 2.2 = (failure is found during the inspection) \cap (the repair is finished at time y) \cap (failure occurs again during $x - y$ period), as shown in Fig. 3(c).

Case 3 The system is in the process of inspection. Case 3 = ($\theta \geq x$), as shown in Fig. 3(d).

Case 1, Case 2 and Case 3 are independent of each other. Therefore, the instantaneous unavailability of the system at time x can be expressed as

$$\bar{A}(x) = \text{Pr}(\text{Case 1}) + \text{Pr}(\text{Case 2}) + \text{Pr}(\text{Case 3}) \quad (2)$$

where $\text{Pr}(\cdot)$ is the probability of each case that causes the system unavailable.

On the basis of assumptions 2) and 5), we can obtain

$$p_1 = \bar{A}(T_i) \quad (3)$$

$$p_2 = \bar{A}(T_i) + \beta \quad (4)$$

where p_1 is the probability that the system is unavailable before the inspection; p_2 is the probability that the system is unavailable if the failure is found during the inspection; β is the probability of failure caused by the inspection.

Thus, the probability of all the cases that cause the system unavailable is as

$$\text{Pr}(\text{Case 1}) = \begin{cases} 0 & x < \theta \\ [A(T_i) + \beta] \bar{M}(x - \theta) & x \geq \theta \end{cases} \quad (5)$$

$$\text{Pr}(\text{Case 2}) = \begin{cases} 0 & x < \theta \\ A(T_i)F(x - \theta) + (\bar{A}(T_i) + \beta) \cdot \int_{\theta}^x m(y)F(x - y)dy & x \geq \theta \end{cases} \quad (6)$$

$$\text{Pr}(\text{Case 3}) = \begin{cases} 1 & x < \theta \\ 0 & x \geq \theta \end{cases} \quad (7)$$

where $M(y)$, $m(y)$ are the distribution function and density function of repair time, respectively; $F(t)$ is the distribution function of failure time; $\bar{M}(\cdot) = 1 - M(\cdot)$; $\bar{A}(\cdot) = 1 - A(\cdot)$.

With Eqs. (1) to (7), the system's unavailability at time x is obtained as

$$\bar{A}(x) = \begin{cases} 1 & x < \theta \\ [\bar{A}(T_i) + \beta] \bar{M}(x - \theta) + A(T_i)F(x - \theta) + [\bar{A}(T_i) + \beta] \int_{\theta}^x m(y)F(x - y)dy & x \geq \theta \end{cases} \quad (8)$$

Let $x = T_i$, $\bar{A}(T_i)$ can be expressed as

$$\bar{A}(T_i) = \frac{\beta[1 - M(T_i - \theta) + \int_{\theta}^{T_i} m(y)F(T_i - y)dy] + F(T_i - \theta)}{M(T_i - \theta) + F(T_i - \theta) - \int_{\theta}^{T_i} m(y)F(T_i - y)dy} \quad (9)$$

The instantaneous availability of the system at time x

can be expressed as

$$A(x) = 1 - \bar{A}(x) \quad (10)$$

If the failure inspection period is T_i , the expected system's availability $A_E(T_i)$ during the i -th inspection period can be calculated by

$$A_E(T_i) = \frac{\int_0^{T_i} A(x) dx}{T_i} \quad i = 1, 2, \dots, N \quad (11)$$

The expected system's availability $A_E(T_d)$ within a finite time t_d can be expressed as

$$A_E(t_d) = \frac{\sum_{i=1}^N \int_0^{T_i} A(x) dx}{\sum_{i=1}^N T_i} \quad (12)$$

In order to make the actual maintenance more convenient and efficient, we use the average value of the age reduction factor within the entire life of the system to replace α_i , as follows:

$$\bar{\alpha} = \frac{\sum_{i=1}^N \alpha_i}{N} \quad (13)$$

All the subsequent inspection periods can be expressed with the first inspection period T_1 , as follows:

$$T_i = \bar{\alpha}^{i-1} T_1, \quad t_i = T_1 + T_2 + \dots + T_i = \frac{1 - \bar{\alpha}^i}{1 - \bar{\alpha}} T_1$$

$$t_d = \sum_{i=1}^N \bar{\alpha}^{i-1} T_1 = \frac{1 - \bar{\alpha}^N}{1 - \bar{\alpha}} T_1 \quad (14)$$

Thus, the objective function and constraints of the availability-based maintenance optimization are as

$$\begin{aligned} & \max A_E(t_d) \\ & \text{s. t.} \quad 0 \leq \sum_{i=1}^N T_i \leq t_d \\ & \quad 0 \leq x \leq T_i \quad i = 1, 2, \dots, N \end{aligned} \quad (15)$$

2 Case Study

Taking 77 of 600 kW wind turbines in a wind farm as the research object, three kinds of key components (i.e., gearbox, generator and spindle) are studied. Based on the data analysis, it is assumed that their failure rate λ and repair rate μ are both exponentially distributed^[16], and the parameters are shown in Tab. 1, where Q is the total number of components monitored, D is the total number of defects per year. Considering the actual situation in maintenance, the inspection time θ is set to be 15 h, the age reduction factor $\bar{\alpha} = 0.95$, and the probability of failure caused by the inspection β is set to be 0.12. The aim is to optimize the availability within the finite time $t_d = 30\,000$ h, which includes inspection time, maintenance time and working time.

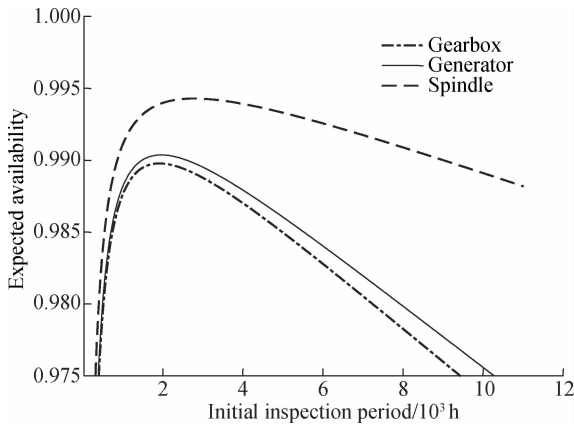
Tab. 1 Parameters of the components

Component	Q	D	λ/h^{-1}	μ/h^{-1}
Gearbox	539	39	8.26×10^{-6}	0.07
Generator	616	40	7.41×10^{-6}	0.06
Spindle	308	7	2.59×10^{-6}	0.05

The optimization results are shown in Tab. 2 and Fig. 4, respectively, where $A_E^*(t_d)$ is the maximum expected availability of the system, T_1^* is the optimal initial inspection period, and N^* is the optimal total number of inspection periods.

Tab. 2 Optimization results

Component	$A_E^*(t_d)/\%$	T_1^*/h	N^*	$T_i = \alpha^{i-1} T_1$
Gearbox	98.98	1 914.9	31	$i = 1, 2, \dots, 31$
Generator	99.04	1 960.2	29	$i = 1, 2, \dots, 29$
Spindle	99.43	2 780.0	16	$i = 1, 2, \dots, 16$

**Fig. 4** The expected availability of the components

From Tab. 2, the maximum expected availability of the above components is above 98%, which meets the requirements of the Chinese National Standard on the wind turbine availability, i. e., 97%^[17]. Through the analysis, it can be concluded that with the increase of the components' failure rate, the optimal inspection period T_1^* should be shortened, and the maintenance number N^* also increases gradually.

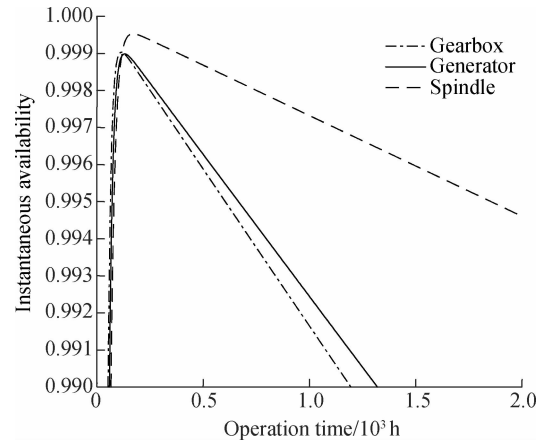
From Fig. 4, we find that the above three curves have a similar trend. The expected availability $A_E(t_d)$ increases rapidly until it reaches the peak, and then it decreases gradually. When T_1 is small, the inspection is frequent, which results in a longer inspection time. When the inspection is carried out, the system is in a state of shut-down. Thus, when T_1 is small, $A_E(t_d)$ is low. With the increase of T_1 , $A_E(t_d)$ increases. The system can achieve maximum availability when the downtime caused by inspection is balanced with the downtime due to failure. As T_1 increases, the system's downtime caused by failure also increases gradually, and it results in a downward trend for system availability. We can also find that the increase of expected availability $A_E(t_d)$ with a low failure rate is faster than that with a high failure rate, i. e., spindle > generator > gearbox. Furthermore, the decline of $A_E(t_d)$

for the component with a low failure rate is obviously slower than that with a high failure rate, i. e., spindle < generator < gearbox. Hence, for a component with a high failure rate, the inspection period should be shortened and the inspection frequency should be increased accordingly.

When the optimal initial inspection period T_1^* is determined, the instantaneous availability under T_1^* is shown in Tab. 3 and Fig. 5, respectively, where $A_{\max}(\psi)$ is the maximum instantaneous availability, and ψ is the time that $A_{\max}(\psi)$ corresponds to.

Tab. 3 Instantaneous availability under T_1^*

Component	$A_{\max}(\psi)/\%$	ψ/h
Gearbox	99.90	115.28
Generator	99.90	130.96
Spindle	99.95	170.15

**Fig. 5** Instantaneous availability under T_1^*

From Tab. 3, for all three components, the maximum instantaneous availability $A_{\max}(\psi)$ is greater than 99.90%. It shows that the model has a good effect on the maintenance optimization for the components. Fig. 5 shows that in the initial stage, the instantaneous availability increases sharply, and it declines after reaching its peak. Since the inspection has achieved a great success in the early stage of the system, the downtime caused by failure decreases greatly, and thus the instantaneous availability is on the rise. With the increase of the system's running time, the system performance degrades gradually. When the inspection and performance degradation reach a balance, system availability is maximized. With the increase of operation time, the degradation of performance plays a dominant role gradually, and it results in the decrease of the availability. In addition, by comparing with Fig. 4, it is found that the variation tendency of the expected availability is similar to the instantaneous availability. All of these increase during the first phase and then they will decline.

Tab. 4 shows the comparison of the optimization results of the two models, i. e., the model based on unequal T_i and the model based on equal T_i . From Tab. 4, the optimal availability has some differences, and the optimiza-

tion result of unequal period inspection is superior to that of equal period inspection. If the model based on equal T_i is used to optimize the availability of the wind turbine's key components, the availability of the components cannot reach the requirements of the Chinese National Standard on the wind turbine availability, 97%. As can be seen from Tab. 4, the two models have advantages. The model based on unequal T_i can make the availability of the system as high as possible, and the number of the inspections of the model based on equal T_i is less, which is beneficial to saving personnel cost. However, for complex electromechanical products such as wind turbines, due to the poor working environment, once a failure occurs, a significant cost will be incurred. It is necessary to increase the availability of wind turbines as much as possible and reduce the occurrence of failure. The effectiveness of the unequal period inspection model is proved.

Tab. 4 Comparison of two models

Component	$A_E^*(t_d)/\%$		T_i^*/h	
	Based on unequal T_i	Based on equal T_i	Based on unequal T_i	Based on equal T_i
Gearbox	98.98	95.42	$1\ 914.9\alpha^{i-1}$ ($i=1, 2, \dots, 31$)	$1\ 840.5$ ($i=1, 2, \dots, 17$)
Generator	99.04	95.81	$1\ 960.2\alpha^{i-1}$ ($i=1, 2, \dots, 29$)	$1\ 930.4$ ($i=1, 2, \dots, 16$)
Spindle	99.43	98.42	$2\ 780.0\alpha^{i-1}$ ($i=1, 2, \dots, 16$)	$2\ 490.7$ ($i=1, 2, \dots, 13$)

3 Conclusions

1) The optimal $A_E^*(t_d)$, N^* and T_1^* can be obtained although the model is complex, and the model proposed in this paper can also be extended to some other products, such as CNC machine tools, automobiles, etc.

2) The research results show that the model based on the unequal inspection period proposed in this paper can improve the availability of the system better than the model based on equal inspection period, and ensure the availability of the wind turbine's key components meet the requirements of the Chinese National Standard for wind turbine availability. It provides a theoretical basis for making a scientific and reasonable maintenance policy for a wind farm.

3) In future study, some other factors can be considered in the maintenance optimization model, including the logistics cost, and spare parts, etc. The bi-objective or multi-objective optimization model can also be considered to achieve better results. In addition, field data obtained from the supervisory control and data acquisition (SCADA) system of wind farms can be integrated into the optimization model.

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基于可用度的非等检测周期和不完全维修优化

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摘要:以多部件系统为研究对象,为提高系统可用度,考虑非等周期检测和不完全维修,建立基于可用度的维修优化模型.引入役龄回退因子描述部件不完全维修效果,提出非等检测周期的建模方法.将系统不可用状态划分为3个相互独立的事件,分析和计算每个事件下的系统不可用度.在分析不可用状态与非等检测周期之间关系的基础上,建立不完全维修条件下的维修优化模型,以优化系统的期望可用度.以某风力机的变速箱、发电机和主轴3个核心部件的维修优化为案例,比较非等周期检测维修模型和等周期检测维修模型的优化结果.结果表明,提出的基于可用度的非等检测周期维修模型能够实时更新风力机系统的维修计划,实现维修活动优化,有效地提高系统可用度.

关键词:役龄回退;检测周期;非等周期维修;维修优化;可用度

中图分类号:TH17