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Mathematical models for properties of mortars with admixtures and recycled fine aggregates from demolished concretes

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Abstract: In order to expand the engineering application of recycle aggregate mortars (RAM) with aggregates from demolished concretes, the models for the properties of RAM and the replacement rate of these recycled fine aggregates were proposed. First, different kinds of mathematical models for the basic properties (compressive strength, water retention rate, and consistency loss) of RAM with two kinds of admixtures, thickening powders (TP) and self-made powdery admixtures (SSCT) designed for RAM, and the replacement rates were established, while the average relative errors and relative standard errors of these models were calculated. Additionally, the models and their error analyses for the curves of drying shrinkage and curing time of RAM + SSCT at different replacement rates were put forward. The results show that polynomial functions should be used to calculate the basic properties of RAM + TP and RAM + SSCT at different replacement rates. In addition, polynomial functions are the most optimal models for the sharp shrinkage sections in the curves of drying shrinkage-curing time of RAM + SSCT, while exponential functions should be used as the models for the slow shrinkage sections and steady shrinkage sections.

Key words: demolished concrete; recycled fine aggregates; admixtures; recycled aggregate mortars; mathematical model
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Last decade, the amount of construction and demolition waste (CDW) generated by the building industries in China accounted for 30% to 40% of the total waste[1]. Therefore, using the recycled fine aggregates (RFA) produced by CDW to replace natural fine aggregate (NFA), can not only reduce the exploitation of natural sand, which may release a significant amount of CO_2 into the environment, but also lower the pollution generated by CDW.

However, the disadvantages of RFA from CDW, such as a high water absorption and high micro powder content, have resulted in poor properties of recycled aggregate mortars (RAM), which have led to severe restrictions on its engineering application[2-3]. Thus, for the sake of high-performance RFA, different strengthening methods were used to lower the water absorption and powder content of RFA[4-9]. Unfortunately, the procedures in the aforesaid methods were long and complex. Therefore, admixtures were used to decrease the adverse effects of the micro powder from RFA on RAM. As a result, the properties of RAM were in accord with the requirements of Chinese specifications[10].

Adopting admixtures to overcome the adverse effects of RFA from CDW on RAM can improve the efficiency during actual engineering application. However, fewer mathematical models for the properties of RAM with admixtures were reported. In this paper, different kinds of mathematical models are used to analyze the relationships between the compressive strength f_{cu}, water retention rate W and consistency loss S of the RAM with two kinds of admixtures, thickening powders (TP) used for normal mortars and powdery admixtures (SSCT) designed for RAM, and the replacement rate R of RFA, respectively. Meanwhile, the models for drying shrinkage and curing time are proposed.

1 Experiments and Mathematical Analyses

1.1 Experiments

P · O 42.5 Portland cement was provided by the Jiangsu Helin Cement Corporation, while the CDW originated from the housings at 1980s was offered by the Jiangsu Zhenjiang Research Institute of Building Science Group Co., Ltd, and the river sand was used as the natural fine aggregate (NFA). Meanwhile, the Zhenjiang Jianbi Power Plants afforded the fly ash (FA), and the mix water stems from laboratory taps. Moreover, the thickening powder (TP) was supplied by the Jiangsu Zhenjiang Research Institute of Building Science Group Co., Ltd, and the SSCT was designed by the authors for the mortars

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with recycled aggregates from CDW.

The mixture proportions are listed in Tab. 1. According to Chinese standard JGJ/T 70—2009, compressive strength was obtained by the 70.7 mm × 70.7 mm × 70.7 mm specimens after being curried for 28 d. Drying shrinkage was obtained from the 40 mm × 40 mm × 160 mm specimens, which were placed in the laboratories at a temperature of (20 ± 2) ℃ and humidity of (60 ± 5)%, every day during the time of 2 to 13 d and 17 to 28 d, and every fifth day during the time of 33 to 53 d and 60 to 90 d. The water retention rate at 2 h was obtained by the fresh mortar specimens. According to Chinese specification GB/T 25181—2010, consistency loss was obtained from the fresh mortar specimens which were put into the environment at a temperature of (23 ± 2) ℃ and humidity of (50 ± 5)%.

Tab. 1 Mixture proportions of mortars at per cubic meters

Mortar type	*m*(cement)/kg	*m*(FA)/kg	*m*(NFA)/kg	*m*(RFA)/kg	*m*(admixtures)/kg	Consistency/mm
RAM + TP	240	60	0 to 1 050	450 to 1 500	20.0	70 to 90
RAM + SSC	240	60	0 to 1 050	450 to 1 500	22.3	70 to 90

1.2 Mathematical analyses

By using the software Excel, exponential functions, linear functions, logarithmic functions, polynomial functions and power functions were used to analyze the relationships between the basic properties (compressive strength, water retention rate and consistency loss) of mortars and the replacement rates. Meanwhile, the models for drying shrinkage-curing time were proposed. Furthermore, the correlation coefficient r, average relative errors δ and relative standard errors e_r of these models were put forward. According to Chinese specification JGJ/T 23—2011, the equations of average relative errors and relative standard errors are as follows:

$$\delta = \pm \frac{1}{n} \sum_{i=1}^{n} \left| \frac{A}{B} - 1 \right| \times 100\% \tag{1}$$

$$e_r = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(\frac{A}{B} - 1 \right)^2} \times 100\% \tag{2}$$

where n is the amount of experimental data; A is the values gained from the experiment for the properties of mortars; and B is the values gained from regression equations for the properties of mortars.

2 Results and Discussion

2.1 Compressive strength and replacement rate of recycled fine aggregates

Fig. 1 plots the experimental values for the compressive strength of mortars at different replacement rates. After calculation, the equations of the exponential function, linear function, logarithmic function, polynomial function and power function for the compressive strength of RAM + TP at different replacement rates are as follows:

$$f_{cu} = 7.970\,2 e^{0.0053R} \tag{3}$$

$$f_{cu} = 0.061\,8R + 7.390\,4 \tag{4}$$

$$f_{cu} = 3.344\,7\ln(R) - 2.262\,5 \tag{5}$$

$$f_{cu} = 0.000\,7R^2 - 0.035\,2R + 10.073 \tag{6}$$

$$f_{cu} = 3.456\,4R^{0.289\,3} \tag{7}$$

Additionally, the equations of the exponential function, linear function, logarithmic function, polynomial function and power function for the compressive strength of RAM + SSCT at different replacement rates are as follows:

$$f_{cu} = 10.264 e^{0.005\,8R} \tag{8}$$

$$f_{cu} = 0.089R + 9.372\,7 \tag{9}$$

$$f_{cu} = 4.93\ln(R) - 4.988\,4 \tag{10}$$

$$f_{cu} = 0.000\,7R^2 - 0.000\,4R + 11.845 \tag{11}$$

$$f_{cu} = 3.977\,9R^{0.324\,4} \tag{12}$$

Furthermore, the equational curves of these equations are shown in Fig. 1, while the regression coefficient and

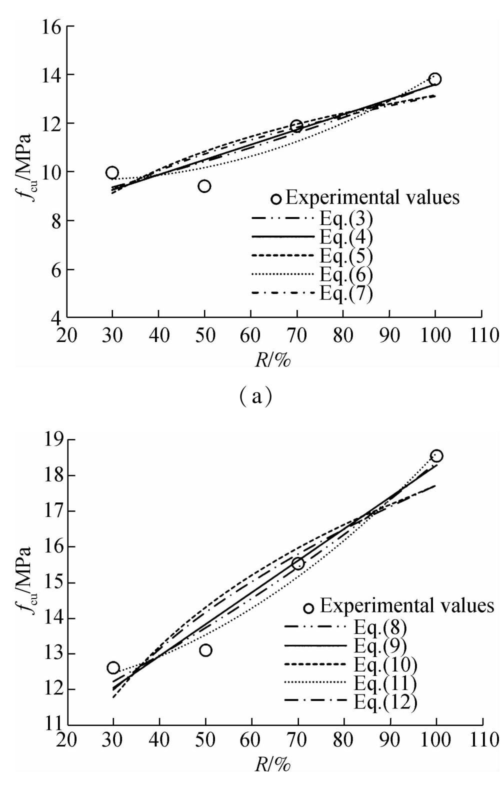


Fig. 1 Experimental values and equational curves of compressive strength of mortars at different replacement rates. (a) RAM + TP; (b) RAM + SSCT

errors for these equations are listed in Tab. 2. It can be seen that the polynomial functions, Eqs. (6) and (11), have the highest regression coefficient and least errors. Hence, they are suggested as the models for the compressive strength of RAM + TP and RAM + SSCT at different replacement rates, respectively.

Tab. 2 Regression coefficient and errors of models for compressive strength of mortars at different replacement rates

Equation		Eq. (3)	Eq. (4)	Eq. (5)	Eq. (6)	Eq. (7)	Eq. (8)	Eq. (9)	Eq. (10)	Eq. (11)	Eq. (12)
Regression coefficient r^2		0.834 7	0.854 0	0.735 1	0.909 8	0.721 4	0.964 6	0.958 4	0.864 8	0.984 0	0.884 4
Error	δ/%	± 5.18	± 5.25	± 7.02	± 4.78	± 6.45	± 2.40	± 2.97	± 5.69	± 1.97	± 4.71
	e_r/%	6.94	7.53	9.73	6.15	8.93	3.3	4.16	7.04	2.53	5.94

2.2 Consistency loss at 2 h and replacement rate of recycled fine aggregates

Fig. 2 plots the experimental values of the consistency loss of mortars at different replacement rates. After calculation, the equations of the exponential function, linear function, logarithmic function, polynomial function and power function for the consistency loss of RAM + TP at different replacement rates are as follows:

$$S = 30.511e^{0.005R} \tag{13}$$

$$S = 0.210\ 9R + 28.819 \tag{14}$$

$$S = 12.046\ln(R) - 6.687\ 8 \tag{15}$$

$$S = 0.000\ 2R^2 + 0.187\ 6R + 29.464 \tag{16}$$

$$S = 13.043R^{0.287\ 3} \tag{17}$$

Additionally, the equations of the exponential function, linear function, logarithmic function, polynomial function and power for the consistency loss of RAM + SSCT at different replacement rates are as follows:

$$S = 9.330\ 1e^{0.011\ 8R} \tag{18}$$

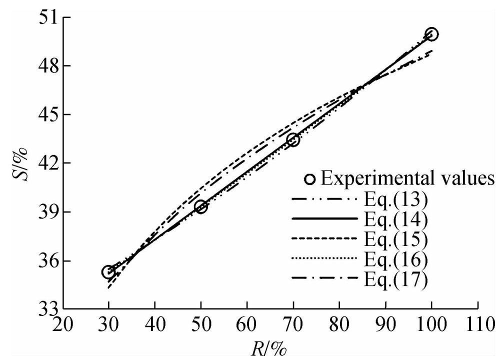
$$S = 0.232\ 5R + 5.913\ 1 \tag{19}$$

$$S = 13.521\ln(R) - 34.205 \tag{20}$$

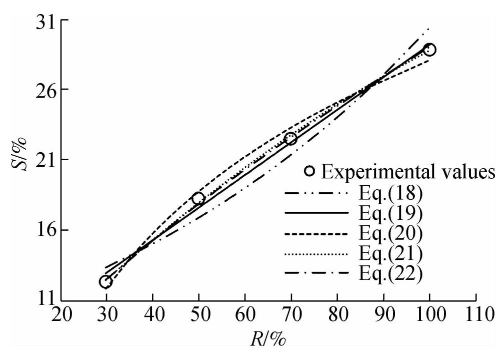
$$S = 0.000\ 8R^2 + 0.340\ 8R + 2.916\ 8 \tag{21}$$

$$S = 1.135\ 6R^{0.703\ 5} \tag{22}$$

Furthermore, the equational curves of these equations are shown in Fig. 2, while the regression coefficient and errors for these equations are listed in Tab. 3. It can be seen that the polynomial functions, Eqs. (16) and (21), have the greatest regression coefficient and least errors. Hence, they are suggested as the models for the consistency loss of RAM + TP and RAM + SSCT at different replacement rates, respectively.



(a)



(b)

Fig. 2 Experimental values and equational curves of consistency loss of mortars at different replacement rates. (a) RAM + TP; (b) RAM + SSCT

Tab. 3 Regression coefficient and errors of models for consistency loss of mortars at different replacement rates

Equation		Eq. (13)	Eq. (14)	Eq. (15)	Eq. (16)	Eq. (17)	Eq. (18)	Eq. (19)	Eq. (20)	Eq. (21)	Eq. (22)
Regression coefficient r^2		0.998 7	0.999 7	0.959 9	1.000 0	0.978 2	0.954 0	0.993 0	0.988 8	0.998 7	0.998 2
Errors	δ/%	±0.46	±0.23	±2.61	±0.23	±1.89	±6.58	±2.74	±3.20	±1.21	±1.11
	e_r/%	0.55	0.28	3.02	0.31	2.20	7.75	3.63	3.78	1.46	1.53

2.3 Water retention rate and replacement rate of recycled fine aggregates

Fig. 3 plots the experimental values of the water retention rate of mortars at different replacement rates. After calculation, the equations of the exponential function, linear function, logarithmic function, polynomial function and power function for the water retention rate of RAM + TP at different replacement rates are as follows:

$$W = 84.713e^{0.000\ 6R} \tag{23}$$

$$W = 0.053\ 1R + 84.66 \tag{24}$$

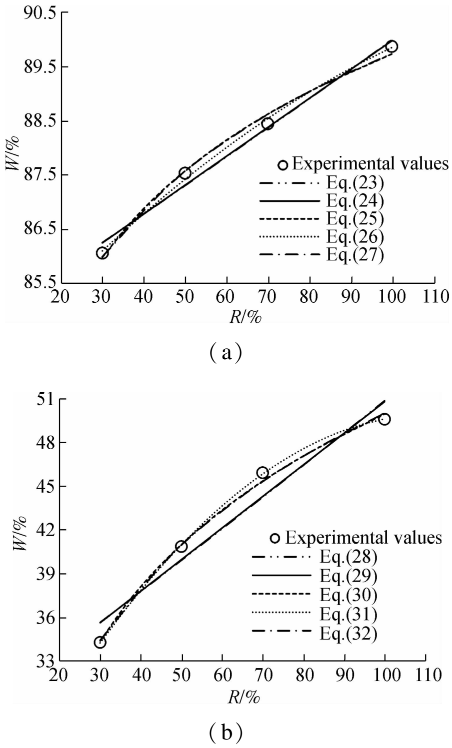


Fig. 3 Experimental values and equational curves of water retention rate of mortars at different replacement rates. (a) RAM + TP; (b) RAM + SSCT

$$W = 3.1013\ln(R) + 75.442 \tag{25}$$

$$W = -0.0002R^2 + 0.0855R + 83.763 \tag{26}$$

$$W = 76.275R^{0.0353} \tag{27}$$

Additionally, the equations of the exponential function, linear function, logarithmic function, polynomial function and power for the water retention rate of RAM + SSCT at different replacement rates are as follows:

$$W = 89.809e^{0.0008R} \tag{28}$$

$$W = 0.0717R + 89.729 \tag{29}$$

$$W = 4.2799\ln(R) + 76.91 \tag{30}$$

$$W = -0.0008R^2 + 0.1732R + 86.92 \tag{31}$$

$$W = 78.346R^{0.0456} \tag{32}$$

Furthermore, the curves of these equations are shown in Fig. 3, while the regression coefficient and errors for these equations are listed in Tab. 4. It can be seen that the polynomial functions, Eqs. (26) and (31), have the greatest regression coefficient and least errors. Hence, they are suggested as the models for the water retention rate of RAM + TP and RAM + SSCT at different replacement rates, respectively.

Tab. 4 Regression coefficient and errors of models for water retention rate of mortars at different replacement rates

Equation		Eq. (23)	Eq. (24)	Eq. (25)	Eq. (26)	Eq. (27)	Eq. (28)	Eq. (29)	Eq. (30)	Eq. (31)	Eq. (32)
Regression coefficient r^2		0.985 5	0.987 1	0.992 4	0.996 8	0.993 5	0.946 0	0.949 2	0.996 2	0.999 8	0.995 7
Errors	δ/%	± 0.17	± 0.17	± 0.12	± 0.24	± 0.11	± 0.47	± 0.45	± 0.10	± 0.13	± 0.11
	e_r/%	0.22	0.21	0.16	0.36	0.15	0.62	0.53	0.14	0.19	0.15

2.4 Drying shrinkage and curing time

Mi et al. [11] suggested that the curves of drying shrinkage-curing time for RAM + TP should be divided into the sharp shrinkage sections (2 to 7 d), slow shrinkage sections (7 to 21 d) and steady shrinkage sections (21 to 90 d). Also, they suggested that the following polynomial function should be used as the mathematical models for sharp shrinkage sections:

$$\varepsilon_{at} = AT^2 + BT + C \tag{33}$$

and the following exponential functions should be used as the mathematical models for slow shrinkage sections and steady shrinkage sections:

$$\varepsilon_{at} = A\ln T + B \tag{34}$$

The experimental values of the drying shrinkage of RAM + SSCT at different curing times are shown in Fig. 4. Meanwhile, after Eq. (33) was used to analyze the sharp shrinkage sections and Eq. (34) was used to analyze the slow shrinkage sections and steady shrinkage sections, the equational curves of the shrinkage-curing time of RAM + SSCT at different replacement rates are depicted in Fig. 4. Furthermore, the parameters, regression coefficient and errors of these equations are summarized in Tabs. 5 and 6.

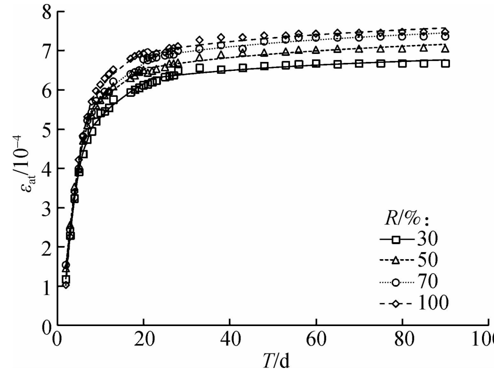


Fig. 4 Experimental values and equational curves of drying shrinkage-curing time at different replacement rates

Tab. 5 The parameters, regression coefficient and errors of models for RAM + SSCT during 2 to 7 d

R/%	Parameters			Regression coefficient	Errors	
	$A_1/10^{-6}$	$B_1/10^{-4}$	$C/10^{-4}$	r^2	δ/%	e_r/%
30	−10.06	1.611 0	−1.635 1	0.999 6	± 0.57	0.74
50	−8.12	1.447 2	−1.080 9	0.995 8	± 1.92	2.53
70	−4.42	1.148 0	−0.604 8	0.998 1	± 1.17	1.47
100	−10.45	1.796 7	−2.159 1	0.999 7	± 0.74	0.99

Tab. 6 The parameters, regression coefficient and errors of models for RAM + SSCT during 7 to 90 d

T/d	R/%	Parameters		Regression coefficient	Errors	
		$A_2/10^{-6}$	$B_2/10^{-4}$	r^2	δ/%	e_r/%
7 to 21	30	123	2.46	0.975 8	± 0.89	1.19
	50	120	2.91	0.969 8	± 0.88	1.11
	70	129	2.79	0.972 6	± 1.19	1.31
	100	134	2.96	0.941 8	± 1.15	1.51
21 to 90	30	33.3	5.261	0.850 4	± 0.79	1.00
	50	41.9	5.270	0.906 4	± 0.85	1.01
	70	43.6	5.482	0.943 8	± 0.61	0.72
	100	43.2	5.628	0.889 8	± 0.95	1.22

3 Conclusions

1) Polynomial functions are suggested as the mathematical models for the compressive strength of the recycled aggregate mortars (RAM) with thickening powders (TP) and powdery admixtures (SSCT), and replacement rates of the recycled fine aggregates from demolished concretes, respectively.

2) Polynomial functions are the most optimal mathematical models for the water retention rate and consistency loss of RAM + TP and RAM + SSCT at different replacement rates, separately.

3) Polynomial functions are suggested to be the mathematical models for the sharp shrinkage sections in the curves of drying shrinkage-curing time for RAM + SSCT, while the exponential functions are recommended to be the models for the slow shrinkage sections and steady shrinkage sections.

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含拆除混凝土再生细骨料与外加剂的砂浆性能的数学模型

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摘要:为了扩大含拆除混凝土再生细骨料砂浆的实际工程应用，建立了再生砂浆性能与其细骨料取代率之间的数学模型. 首先，分别提出了含稠化粉与自制粉状外加剂的再生砂浆基本性能(抗压强度、保水率与稠度损失)与其细骨料取代率之间的多种数学模型,并计算其平均相对误差与相对标准误差. 其次，建立了不同细骨料取代率下含粉状外加剂的再生砂浆干缩值与龄期之间的数学模型,并进行了相应的误差分析. 结果表明：含稠化粉与粉状外加剂的再生砂浆基本性能与其细骨料取代率之间的数学模型均宜采用多项式函数;不同细骨料取代率下含粉状外加剂的再生砂浆干缩-龄期曲线中急缩段的数学模型宜采用多项式函数，缓缩段与稳缩段的数学模型宜采用指数函数.

关键词:拆除混凝土；再生细骨料；外加剂；再生砂浆；数学模型

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