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Design and implementation of square Helmholtz coil for uniform magnetic field generation

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Abstract: In order to calibrate electrical instruments and generate a constant magnetic field, a novel design method for square Helmholtz coil is proposed. According to the superposition principle in electromagnetics, the theory of the square Helmholtz coil is established, and the design method is verified by Matlab calculation. Compared with conventional circular Helmholtz coil, the novel square one is with a larger uniform region. Simulation work is conducted in Maxwell, and the distribution of the magnetic field is obtained. The results demonstrate the validation of the applied calculation method of the proposed Helmholtz model. The space utilization rate η is used to make a comparison between the square and circular coils for the uniform region. The square Helmholtz coil is fabricated, the length of a single square coil is 1.5 m, and the amplitude of the magnetic field is controlled by the current. The GSM-19T proton magnetometer is used to measure the amplitude of the magnetic field generated by the square Helmholtz coil. Experimental results indicate that a wide-range variable uniform magnetic field from 0 to 120 μT is generated in the center of Helmholtz coils.

Key words: proton magnetometer; square Helmholtz coils; space utilization rate; uniform magnetic field

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Uniform magnetic field is important in scientific experiments, which can calibrate the measuring range of magnetometers. The uniform magnetic field is usually applied to superpose with other magnetic fields, such as earth's magnetic field. In practice, the dimension of the uniform field needed by application sometimes reaches some hundred centimeters. The scale of the uniform magnetic field zone is worth investigating. Furthermore, the generation method of the uniform magnetic field is also of importance. Variable functions are introduced in Refs. [1 −3].

The coil system together with the current source may be the simplest way to generate uniform magnetic fields. Typically, there are round coils, solenoid coils, and spherical coils catering for the requirements of various specific cases[3]. Among these coils, the circular standard Helmholtz coil is frequently used by the magnetic field generator for its field uniformity in the center of the coils[4−9]. Circular coils positioned parallel with each other can generate a uniform magnetic field in the center of the coils. However, the scale of the uniform magnetic field zone is very limited when compared with experimental facilities. The space utilization rate of a circular Helmholtz coil is low, and it restricts the further application with large volumes of uniform magnetic field requirement. The space utilization rate has many mathematical expressions. This paper presents the design of square Helmholtz coils, and the space utilization rate is defined as the ratio of the volume of the desired uniform magnetic field region to the volume of the generating system occupied by coils. If both coils are too large, the inductance and resistance of the coils will significantly increase, which needs a larger power to feed, and it wastes more electric energy. Meanwhile, more consumption of the material will also increase the cost.

The design and implementation of the square Helmholtz coil is presented, which can obtain a large uniform magnetic field area. The calculation results show that the provided uniform magnetic field is determined by the parameters of coils and the amplitude of the current. Simulation results are well matched with the electromagnetic theory based on calculation. In order to research the performance of the designed calculation method about the magnetic field generator, a square Helmholtz coil is fabricated, the arm of the coil is 0.75 m and the maximum current in the coil is 4.6 A. The GSM-19T proton magnetometer is used to measure the dynamic range of the Helmholtz coil. Experimental results show that the magnetic field in the square Helmholtz coil is uniform, and the amplitude of the magnetic field can be varied by the input current, which makes it a variable candidate for uniform magnetic field generation.

1 Structure Design and Analysis

1.1 Coil structure and function

The traditional circular Helmholtz coil generates the uniform magnetic field in the center of the structure. The distance between two arms is equal to the radius of the coil, and the current in the coil is in the same direc-

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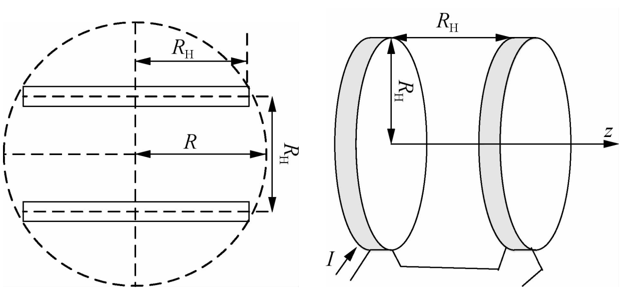
tion[8-9]. The schematic diagram is sketched in Fig. 1. However, the volume of the constant magnetic field in center is limited, and the geometry of the coil is difficult to fabricate. Based on the circular Helmholtz theory[10-12], a novel square Helmholtz coil is designed, as shown in Fig. 2.



(a) (b)

Fig. 1 Layout of circular Helmholtz coil. (a) Side view with the dimensions of numerical model; (b) Operation prototype for the magnetic field generator

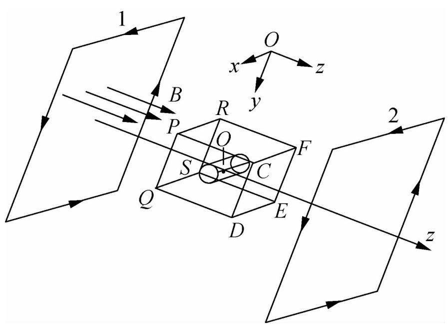


Fig. 2 Operation model of square Helmholtz coil

The uniform magnetic field region (PQRS-CDEF) is shown in Fig. 2. The calculation of the proposed model of the Helmholtz coil is based on the superposition theory in electromagnetics. The generated magnetic field is superposed by eight coils with current in them. The amplitude of the magnetic field can be varied by stimulated currents. The proton magnetometer is used to measure the uniform region of the magnetic field generated by Helmholtz coils. The availability of square Helmholtz coils can be demonstrated by the measurement of proton magnetometers, while the parameters of coils should be discussed.

1.2 Helmholtz coil theory and calculation

The magnetic field produced by a current-carrying wire in free space is expressed as

$$B = \frac{\mu_0 I}{4\pi d}(\cos\theta_1 - \cos\theta_2) \tag{1}$$

where $\mu_0 = 4\pi \times 10^{-7}$ N/A^2 is the vacuum permeability; I is the current in coil; θ_1, θ_2 are the angles from two end points of the arm to point P, as shown in Fig. 3. The direction of the magnetic field and current is followed by the Ampere rule. The amplitude of the magnetic field produced by a single coil can be calculated by superposing vectors of four arms[5]. The principle is shown in Fig. 3.

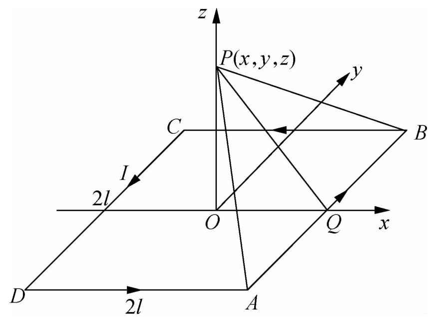


Fig. 3 Magnetic field produced by a square Helmholtz coil

When P is in the z axis, the magnetic field is expressed as

$$d = \sqrt{l^2 + z^2} \tag{2}$$

$$\cos\theta_1 = \cos\angle PAQ = \frac{QA}{PA} = \frac{l}{\sqrt{2l^2 + z^2}} \tag{3}$$

$$\cos\theta_2 = -\cos\angle PBQ = \frac{QB}{PB} = -\frac{l}{\sqrt{2l^2 + z^2}} \tag{4}$$

$$B_{AB} = \frac{\mu_0 I l}{2\pi \sqrt{l^2 + z^2}\sqrt{2l^2 + z^2}} \tag{5}$$

Similarly, $B_{BC} = B_{CD} = B_{DA} = B_{AB}$. Due to the symmetrical property of square, the direction of the parallel component of the magnetic field in the z axis is opposite, and the vertical component is superposed by

$$B = 4B_{AB}\cos\angle PQO = \frac{2\mu_0 I l^2}{\pi(l^2 + z^2)\sqrt{2l^2 + z^2}} \tag{6}$$

According to the Ampere and Biot-Savart law (AB method), the magnitude of the magnetic field in the Helmholtz coil is expressed as

$$B = \frac{2\mu_0 I l^2}{\pi[l^2 + (a+z)^2]\sqrt{2l^2 + (a+z)^2}} + \frac{2\mu_0 I l^2}{\pi[l^2 + (a-z)^2]\sqrt{2l^2 + (a-z)^2}} \tag{7}$$

The origin of the coordinate is at the center of two square coils in the above equations. The amplitude of the magnetic field around the origin of the coordinate is discussed, and magnetic field B is expanded based on the Taylor equation.

$$B(z) = B(0) + z\left(\frac{\partial B}{\partial z}\right)_{z=0} + \frac{z^2}{2}\left(\frac{\partial^2 B}{\partial z^2}\right)_{z=0} + \frac{z^3}{3!}\left(\frac{\partial^3 B}{\partial z^3}\right)^3_{z=0} + \frac{z^4}{4!}\left(\frac{\partial^4 B}{\partial z^4}\right)^4_{z=0} + \Lambda \tag{8}$$

The magnetic field B is origin-symmetric, $B(z) = B(-z)$, B is even functioned, and the coefficient of the odd phase in expansion is therefore offset as

$$B(z) = B(0) + O(z^2) + O(z^4) + \Lambda \tag{9}$$

if $O(z^2) = \frac{z^2}{2}\left(\frac{\partial^2 B}{\partial z^2}\right) = 0$, $O(z^4)$ is the 4th order of the equation and it can be ignored, then the uniform magnetic field in the center is obtained. The second partial derivative is functioned to $B(Z)$. After calculation, we can obtain

$$6a^6 + 18n^4 + 11n^2 - 5 = 0 \quad (10)$$

Suppose that $a = nl$ and the unique solution is obtained, $n = 0.544\,5$. At the same time, the magnetic field is uniform in the coil center, and the optimal distance between two coils is $2a = 1.089l$. The amplitude of the magnetic field is obtained by

$$B_0 = 0.648\,1 \frac{\mu_0 NI}{l} \quad (11)$$

where N is the turns of coils. In order to analyze the uniform region of the magnetic field in the center coil, the deviation rate δ of the magnetic field is introduced,

$$\delta = \left| \frac{B(x,y,z) - B(0,0,0)}{B(0,0,0)} \right| \quad (12)$$

where $B(x, y, z)$ is the point around origin. Matlab is used to calculate the deviation. The results show that the uniform magnetic field range is $-0.19l \leqslant z \leqslant 0.19l$, $-0.222l \leqslant x \leqslant 0.222l$, $-0.222l \leqslant y \leqslant 0.222l$, when $\delta \leqslant 0.1\%$.

As shown in the calculation results, the square Helmholtz coil performs a larger uniform magnetic field than the traditional circular one. Also, the uniform magnetic field can be generated for electromagnetic applications. The sensor of the magnetometer can be arranged into the uinform region, and the calibration measurement can be conducted by simply controlling the current in coils.

2 Simulated and Measured Results

The square Helmholtz structure can generate a uniform magnetic field in the center of its coils according to the calculation results. In order to study the field distribution and the amplitude of the magnetic field around origin, Matlab is used to establish the Helmholtz model. According to the relationship between the amplitude of the magnetic field and parameters of coils, the length of the arms is designed to be $2l = 1.5$ m, the turns of the coils and the maximum currents are, respectively, $N = 20$ and 4.6 A. The calculation results are shown in Fig. 4. In this design, the variable range of the magnetic field from 0 to 120 μT can be theoretically obtained.

Fig. 4(a) shows the 2D magnetic field distribution in the horizontal plane, where $Z = 0$. It can be seen that the magnetic field is uniform at the center of coils. Fig. 4(b) shows the distribution of the magnetic field along the Y axis ($X = 0$, $Z = 0$) and the amplitude of the magnetic field is 100 μT. Fig. 4(b) is the front view of Fig. 4(a), which is in the Y direction. The calculation results confirm the availability of the square Helmholtz coil. However, the uniform region is limited to inside the coils, as shown in Fig. 4. When the reference point is next to the edge of the coil, the magnetic field has a nonlinear tendency, and the field is no longer a constant. The uniformity characteristics can be described by Eq. (12).

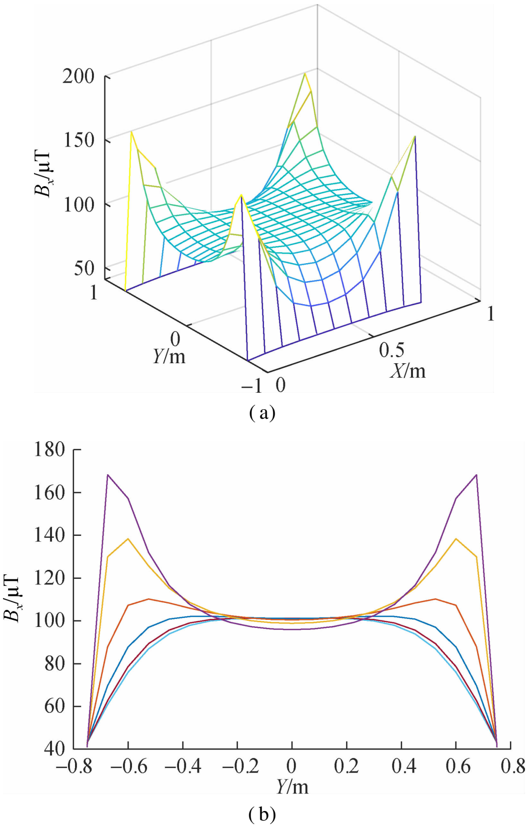
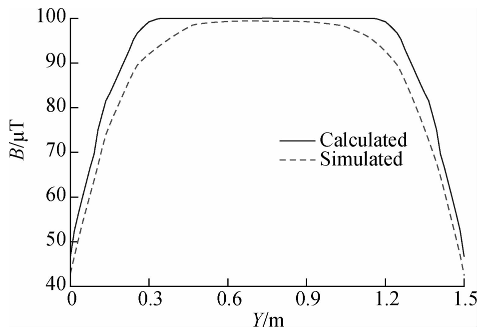


Fig. 4 Calculation results in Matlab. (a) Magnetic field distribution in *XOY* plane; (b) Amplitude of magnetic field along *Y* axis in *XOY* plane

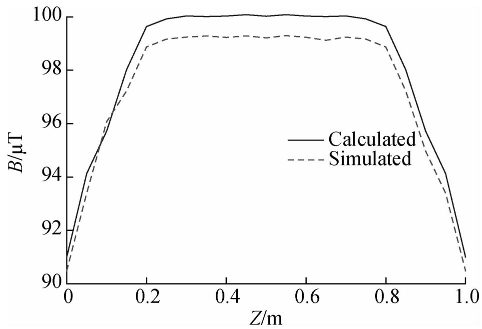
Furthermore, Ansoft Maxwell software is used to simulate the designed Helmholtz coil. Parameters are the same as the calculated ones based on above discussion. Comparisons between the calculated and simulated results are shown in Fig. 5. The amplitude of the magnetic field along the Y axis is depicted in Fig. 5(a). Fig. 5(b) illustrates the same relationship but in the vertical direction. The simulated results are slightly reduced, the deviation is primarily caused by limited region for calculation in Maxwell, and with the increase of the volume, curves are much closer to the calculated results.

Simulation work concerning 2D distribution of the magnetic field in the Helmholtz coil is also conducted. The model of the circular Helmholtz coil is designed as a comparison to the square one. Vectors and the magnitude intensity of the magnetic field are shown in Figs. 6(a) and (b). The square Helmholtz shows a similar uniform property to the circular coils, and follows the above calculation theory. In contrast, the model and simulation results of the circular Helmholtz coils are shown in Fig. 1 and Fig. 6(a). The distance between two coils has the same dimension of the radius of the coil. The amplitude of the magnetic field is expressed as

$$B = \left(\frac{4}{5}\right)^{2/3} \frac{\mu_0 nI}{R} \quad (13)$$

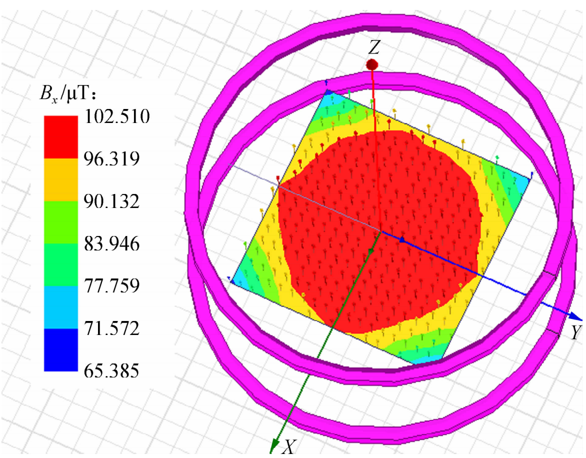


(a)

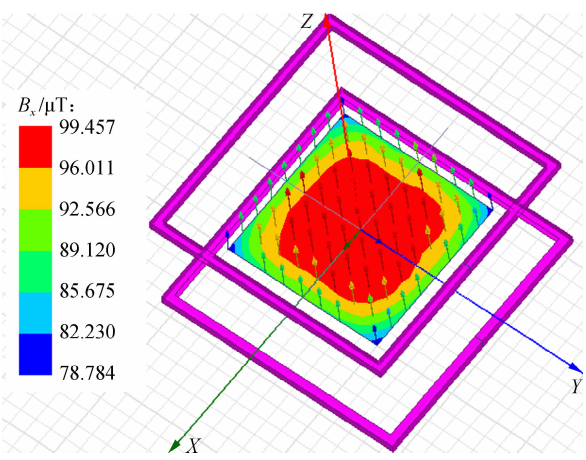


(b)

Fig. 5 Comparison between calculated and simulated results of magnetic field. (a) Y axis; (b) Z axis



(a)

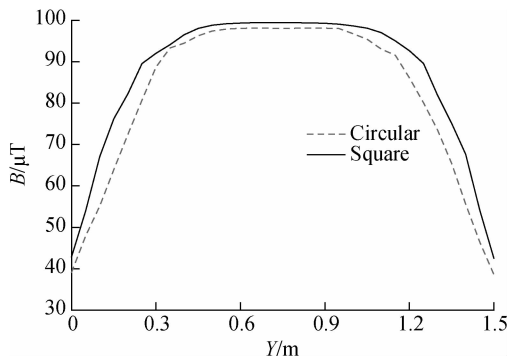


(b)

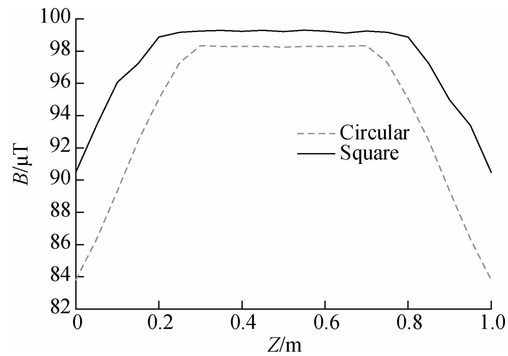
Fig. 6 2D simulation results of magnetic field. (a) Circular coils; (b) Square coils

$$\eta = \frac{V_{\text{uniform region}}}{V_{\text{coils}}} \tag{14}$$

In order to evaluate the performance of Helmholtz coils, the space utilization rate η is proposed to compare the square Helmholtz coil with the circular one, as depicted in Eq. (14). The volume of the proposed square Helmholtz coil is $V_S = 1.836\ \text{m}^3$ ($l = 1.5$ m, $2a = 0.816$ m). As for circular coils, the radius is supposed to be 0.836 m when V_C is $1.832\ \text{m}^3$. Simulations are conducted in Maxwell and comparisons in different shapes are shown in Fig. 7. Calculated by Eq. (12) in Matlab, the uniform region (for example $\delta = 1\%$) is obtained for the two types of coils. The space utilization rate η for the square Helmholtz coil and circular Helmholtz coil are, respectively, 17.94% and 4.36%. The square shape of the coil can effectively increase the uniform region of the magnetic field.



(a)



(b)

Fig. 7 Comparison of magnetic field between circular and square coils with the same volume. (a) Y axis; (b) Z axis

In order to validate the availability of the proposed square Helmholtz coil, the structure with wires is fabricated, and the parameters follow the above discussion. The length of the arms is $2l = 1.5$ m, the distance between coils is 0.816 m, and the turns of wires are 20. The standard of the copper coil is $S = 2.5\ \text{mm}^2$ and thus the resistance of a single coil is 1.68 Ω. Variable power resistance is prepared for experiments, and the amplitude of the magnetic field varies with the current in the coil.

The GSM-19T proton magnetometer designed by the GEM company is applied to measure the amplitude of the magnetic field. In the experiment, the current in the coil is controlled so that the generated magnetic field can vary from 0 to 70 μT, while the earth's magnetic field is about 50 μT, and the fields will be superposed in the center of the coils. The vector direction of the generated magnetic field is varied with the input current, and it can be in the same or opposite direction compared with earth magnets. A variable magnetic field from 0 to 120 μT can be obtained. Experimental results are given in Tab. 1 and the corresponding measurement process is shown in Fig. 8.

Tab. 1 Measurement results of magnetic field

Resistance	Current direction	Field state	Magnetic field/μT
Series 3 Ω	$X \to Y$	Superpose	81
Series 2 Ω	$X \to Y$	Superpose	88
Series 1 Ω	$X \to Y$	Superpose	99
Series 0.5 Ω	$X \to Y$	Superpose	107
Series 0 Ω	$X \to Y$	Superpose	120
Series 50 Ω	$Y \to X$	Offset	34
Series 20 Ω	$Y \to X$	Offset	28
Series12 Ω	$Y \to X$	Offset	20
Series 10 Ω	$Y \to X$	Offset	19
Series 8 Ω	$Y \to X$	Offset	16

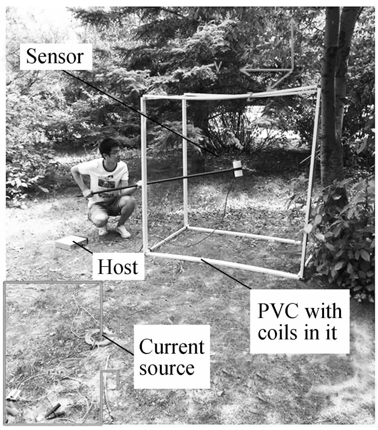


Fig. 8 Experimental process of the magnetic field measurement with the GSM-19T proton magnetometer

3 Conclusion

The design and implementation of the square Helmholtz coil is presented in this paper. Simulation results indicate that the square Helmholtz coil has a uniform region of the magnetic field, which is larger than that of the circular one. The square Helmholtz coil is fabricated in the experiment and the generated magnetic field is measured by the GSM-19T proton magnetometer. The results are well matched to the design. The magnetic field can be varied by the current, which makes it a suitable device for calibration of electrical devices and fundamental physical experiments. However, the proposed design has not realized the arbitrary direction of a uniform magnetic field. Further research of combined coils will be carried out to fulfill the issue.

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用于激励匀强磁场的方形 Helmholtz 线圈设计与实现

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摘要: 为了标定电子仪器设备,产生稳恒的磁场,提出了一种新颖的方形 Helmholtz 线圈的设计方法. 根据电磁场叠加原理,建立了方形 Helmholtz 线圈理论模型,并用 Matlab 软件验证了模型的正确性. 该模型较传统圆形 Helmholtz 线圈有更大的匀强场区. 利用 Maxwell 软件进行仿真计算,得到了线圈周围磁场的分布,仿真结果验证了 Helmholtz 线圈计算方法的正确性. 用空间利用率 η 比较了方形线圈与传统圆形线圈所产生的匀强磁场范围. 制作了方形 Helmholtz 线圈,线圈臂长 1.5 m,匀强磁场的大小由输入电流控制. 应用 GSM-19T 型质子磁力仪测量方形 Helmholtz 线圈激励磁场的幅值. 实验表明,方形 Helmholtz 线圈中心可产生 0 ~ 120 μT 幅值可调节的均匀电磁场区.

关键词: 质子磁力仪; 方形 Helmholtz 线圈; 空间利用率; 匀强磁场
中图分类号: TN80