

# Combination advertising with budget constraint

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**Abstract:** An appropriate advertising mode selection and allocation of advertising budgets considering advertising budget constraints are developed with the game theory and optimization model. The results show that the advertising budget and the weakening factor of the online advertising effects on the traditional advertising affect sellers' advertising, pricing strategies and budget allocation. The effectiveness of combination advertising is a Pareto optimum relative to the traditional advertising in a certain range. The weakening effect of the online advertising on the traditional advertising has a nonlinear effect on the advertising strategy and pricing strategy. In addition, sellers choose combination advertising with different budget constraints, and there is an optimal budget allocation ratio. In certain cases, sellers can obtain an optimal advertising expenditure less than the budget constrain. When sellers adopt combination advertising with enough budget, they do not invest in traditional advertising without limit. Moreover, compared with enough budget, when sellers adopt combination advertising with budget constrain, sellers do not decrease the advertising allocation of online advertising since online advertising is more efficient.

**Key words:** combination advertising; budgets allocation; traditional advertising; online advertising; budget constraint

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With the development and popularization of the Internet, online advertising has replaced traditional TV advertising, and has become the most widely used form of advertising. Some analysis data shows that users spend more time on the Internet from 146 min in 2010 to 200 min now per day, while the time spent on traditional channels, such as TV, newspapers, radios, has dropped from 176 to 152 min. The rapid growth of online advertising has compressed the traditional advertising markets quickly. The traditional advertising marketing scale is shrinking day by day, and it results in fewer market shares, advertising incomes and profits. The phenomenon indicates that online advertising is gradually eroding the

market share of traditional advertising, so that the view of "traditional advertising is dead" emerged. Has traditional advertising really lost its value and will it be completely replaced by online advertising? According to the advertising statistics released by CTR and other research institutes, despite the rapid development of online advertising, traditional advertising represented by TV advertising still occupies more than 50% market share and advertising budget. Currently, traditional advertising still occupies an important advertising market position. Compared with online advertising, traditional advertising has the advantages of a wider dissemination scope, longer advertising cycle, cultivating potential consumers, and attraction of consumers' attention to products and so on. Hence, online advertising cannot replace traditional advertising completely within a limited time, and traditional advertising still has a relatively strong vitality. Traditional advertising and online advertising will still coexist for a long time. In particular, many sellers, regardless of their market position (monopoly or competition), usually do not take a single advertising mode (traditional advertising or online advertising) when conducting advertising promotion, they may use traditional advertising and online advertising simultaneously (combination advertising). For example, when Apple promotes a new iPhone, it targets mail to special customers and advertises on TV simultaneously. Hence, sellers can choose traditional advertising, online advertising and combination advertising to market their products. Therefore, how to choose the advertising mode has become one of the basic issues that enterprise marketing decision-makers need to focus on.

In addition, sellers do not only need to consider advertising effectiveness, but are also subject to the constraint of the advertising marketing cost budget during the sellers' advertising promotion. Sellers need to have a reasonable budget (which they can afford) to maximize their interests.

The existing literature mainly focuses on the difference between online advertising and traditional advertising. Current research suggests that online advertising obtains higher advertising efficiency than traditional advertising. Meanwhile, it can reduce advertising costs. Online advertising uses customers' information to map customers, and predicts their demands. If the demands can be archived, sellers may target advertising to the potential customers. Sometimes, we call online advertising targeted advertising as well. This research focuses on how companies use tar-

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geted advertising to differentiate potential customers and reduce advertising resources waste. Sellers can use targeted advertising combined with targeted pricing to achieve higher revenues<sup>[2-4]</sup>. Targeted advertising seems to be better than traditional advertising. Iyer et al.<sup>[5-7]</sup> compared targeted advertising and traditional advertising under competitive conditions. They showed that targeted advertising can help mitigate competition among sellers and enable sellers to obtain more benefits. However, the research of Refs. [8–10] showed that in some certain situations, the actual effectiveness of online advertising is not better than traditional advertising, and may hurt the sellers' interests. Although online advertising keeps the advantages of higher efficiency and lower cost, it cannot completely replace traditional advertising. Recently, research is not limited to the comparison of online advertising and traditional advertising, and it began to focus on a combination of them. Liaukonyte<sup>[11]</sup> showed that TV advertising impacts the search behavior of the product and ultimately increases the sales. Joo et al.<sup>[12-13]</sup> demonstrated the view by empirical studies. Sridhar et al.<sup>[14]</sup> showed that traditional advertising and targeted advertising can harm each other when used separately, but sellers can achieve higher returns with using combined advertising. de Haan et al.<sup>[15]</sup> verified that a variety of advertising model combinations can achieve better results with modelling and empirical methods.

Sellers' advertising strategies are constrained by the sellers' advertising budget. How sellers develop advertising budgets is a key strategy for sellers to conduct marketing activities. Dorfman et al.<sup>[16]</sup> used a model to analyze the link between advertising budgets and optimal advertising decisions, prices, and product quality. Wang et al.<sup>[17]</sup> gave an effective budget allocation algorithm for multi-channel advertising. Wright<sup>[18]</sup> designed the theorem for obtaining the optimal advertising budget and verified it. Kong et al.<sup>[19]</sup> designed a new method to allocate an advertising budget which can achieve the best performance for the advertiser. de Haan et al.<sup>[15]</sup> used a combination of modelling and empirical methods to discuss different advertising models and the allocation of advertising budgets under different sale channels. Zia et al.<sup>[20]</sup> empirically studied the optimal strategy of advertising budget allocation under multi-platform delivery of online advertising.

Considering that there are few theoretical studies on the application of the combination of online advertising and traditional advertising in the current research, we attempt to use mathematical models to explain the impact of advertising budget constraints on combination advertising strategies.

## 1 Model Setup

With the rapid development of e-commerce, sellers may benefit from the monopoly ability but face more in-

tensive competition. For instance, in the beer market of Jiangsu, CR Snow has a market share over 60%, while the market share of Tsingtao Beer which takes the second share is less than 10%. It has formed a regional monopoly for CR Snow. But in the digital era, small sellers may seize the monopoly sellers' market share more easily. So monopoly sellers need to maintain or expand their market share with a variety of marketing tools. As one of the most important marketing tools, advertising attracts sellers' attention. With the Internet, sellers can use traditional advertising and online advertising simultaneously for marketing.

We consider a retailer channel in the market: a monopoly seller and potential customers. Customers can obtain the product information by traditional advertising. Meanwhile, sellers cannot obtain customers' privacy to deliver online advertising without traditional advertising. So, sellers can choose two different modes of advertising: traditional advertising or the combination of traditional advertising and online advertising. Also, sellers set the price and advertising strategies with advertising budgets. The basic demand follows the negative linear demand curve, that is, when the product's price is higher, the demand decreases linearly with price  $p$ . The basic demand of the market without advertising is  $a - bp$ , where  $a$  shows the fundamental demand, and  $b$  represents the price elasticity.

Sellers set prices and advertising strategies to maximize their revenue. Traditional advertising cannot recognize potential customers precisely and cannot target advertising to them. Potential customers receive advertising indistinctively. Customers with uniform distribution receive the traditional advertising with the same possibility. Online advertising can recognize customers based on traditional advertising and can target advertising to them precisely. We assume that all of the advertising modes can raise the seller's demand. Sellers invest  $A$  to obtain an increase demand of  $m$  units with traditional advertising. The traditional advertising cost is linearly related to the amount of customers ( $\varphi$ ) receiving the advertising. The traditional advertising cost can be given as  $C(\varphi) = \varphi A$ . Different from traditional advertising, the increasing demand of online advertising is related to the online advertising scope  $\varphi$ . The unit cost of online advertising is  $B$ , and it can bring the sellers increasing demand  $t$  at most. Following the incremental marginal cost, online advertising cost can be given as  $C(\varphi) = \varphi^2 B/2$ .

Customers in the market receive traditional advertising indistinguishably, and online advertising can target special customers individually. When consumers receive the advertising, they may respond to the advertising with a variety of follow-up behaviors. Some of them may ignore the advertising and no purchase intention occurs. Some of them are interested in the advertised product and purchase

immediately. In some cases, customers are partly interested in the advertising product and search for the product's information to make the purchase decisions. When the advertising receivers take further steps in searching for more information about the product, the sellers can target them using online advertising to promote their willingness to pay. Sellers may deliver online advertising and traditional advertising simultaneously, and there may be repeated advertising deliveries, thus resulting in lower unit utility of advertising due to the substitution effect. In this paper, online advertising collects customers' information based on traditional advertising, so the substitution effect mostly is online advertising on traditional advertising. The negative effect of online advertising on the traditional advertising market is negatively correlated with the investment of online advertising. We define  $\lambda$  to be the weakening factor of online advertising to traditional advertising, where  $0 \leq \lambda < 1$ . Without an advertising budget constraint, traditional advertising covers the whole market with an increasing demand  $m$ , and online advertising is adopted with an expected demand increment  $t$ . The gross demand is  $\lambda m + t$ .

## 2 Analysis of Advertising Strategies without Budget Constraint

### 2.1 Traditional advertising strategies

Without an advertising budget constraint, sellers should decide whether to deliver advertising first. Sellers want to maximize their profits. As for the former assumption, sellers should invest in traditional advertising first to allow the market to obtain the product's information. Sellers invest in traditional advertising when they can obtain more profit with advertising. We can derive the following proposition.

**Proposition 1** Sellers invest in traditional advertising if the traditional advertising conversion rate satisfies  $\frac{m}{A} > \frac{4b}{2a+m}$ . Sellers' optimal price with traditional advertising is  $\frac{a+m}{2b}$ , and their optimal profit is  $\frac{(a+m)^2}{4b} - A$ .

Proposition 1 shows that the traditional advertising conversion rate affects sellers' advertising strategies. Traditional advertising effectiveness is important to sellers. Different from online advertising, the traditional advertising conversion rate determines how many potential customers will buy the product. A higher conversion rate gives more payoffs of the advertising investment. Meanwhile, the market structure affects the advertising strategy, and sellers need to consider the basic market and price elasticity. We can have the following corollaries.

**Corollary 1** When sellers adopt traditional advertising, traditional advertising should have a lower advertising conversion rate with a larger basic market. Mean-

while, traditional advertising should have a higher conversion rate with a greater price elasticity.

Corollary 1 shows the connection between the advertising conversion rate and basic market's properties. A large basic market can guarantee sellers' profit, and sellers' investment in traditional advertising raises the price and decreases the basic demand, but a larger basic demand can bear the loss of basic demand more since advertising may obtain more profit. Sellers need to investigate the basic market's properties as references when they adopt traditional advertising. As a consequence, we have the following corollary.

**Corollary 2** When sellers adopt traditional advertising, the price elasticity of the basic market satisfies  $b < \frac{2am+m^2}{4A}$ .

The price elasticity cannot be more than  $\frac{2am+m^2}{4A}$ . A greater elasticity makes the market more unstable, when sellers choose to use advertising, customers' sensitivity to price of the product is higher than that without advertising. The reduction of demand caused by a higher price elasticity is more than the increment of demand caused by advertising.

**Corollary 3** The demand increment by the traditional advertising is  $m \geq a \left( \sqrt{\frac{4bA}{a^2} + 1} - 1 \right)$ . Meanwhile, advertising unit cost is  $A < \frac{3a^2}{4b}$ , and sellers adopt traditional advertising.

Advertising cost is one of the major factors that sellers should consider first. If the unit advertising cost is over  $\frac{3a^2}{4b}$ , sellers' marginal revenue will be less than 0, and they may not invest in traditional advertising. Meanwhile, the minimum increment cannot be less than  $a \left( \sqrt{\frac{4bA}{a^2} + 1} - 1 \right)$ , that is, when sellers deliver advertising to the whole market, they need to attract more than  $a \left( \sqrt{\frac{4bA}{a^2} + 1} - 1 \right)$  to purchase the product. Only with these two conditions, can sellers benefit from traditional advertising. In practice, Corollary 3 is meaningful to the new product. At the beginning, the producer of new product should create the basic demand at the first step to gain a large increment with traditional advertising as much as they can.

### 2.2 Combination advertising strategies

Sellers obtain more advertising options for marketing with online advertising. In this section, we compare traditional advertising and combination advertising. If sellers can obtain more payoff by investing on online advertising based on traditional advertising, they will invest in com-

bination advertising. Assuming that online advertising can target the potential customers perfectly, the one who receives online advertising will buy the advertised products. On the other hand, online advertising may be targeted to those whom have already received traditional advertising and purchased the product. Online advertising weakens traditional advertising effectiveness. On the other hand, online advertising may be targeted to the potential customers who have received the traditional advertising but not purchased yet, and online advertising increases their willingness to purchase. Then, online advertising is complementary to traditional advertising.

With combination advertising, sellers should invest both in traditional advertising and online advertising. Sellers should cover the market with traditional advertising as possible as they can, so they will invest  $A$  in traditional advertising. But should they invest in online advertising, if so, how much should they invest? Without budget constraints, the optimal combination advertising strategy is characterized by proposition 2.

**Proposition 2** When sellers adopt combination advertising, the optimal price is  $\frac{a}{2b} + \frac{\lambda m}{2b} + \frac{a + \lambda m}{2b(2Bb - 1)}$ , and sellers' maximum revenue is  $\frac{B(a + \lambda m)^2}{2(2Bb - 1)} - A$ ; online advertising covers  $\frac{a + \lambda m}{2Bb - 1}$  of the market, and the optimal demand is  $\frac{a}{2} + \frac{\lambda m}{2} - \frac{a + \lambda m}{2(2Bb - 1)}$ .

Based on Proposition 2, we can obtain the following corollary.

**Corollary 4** If sellers adopt combination advertising, the optimal demand does not always increase as the weakening factor increases. Also, sellers set a higher optimal price and obtain more profit with a larger weakening factor.

Corollary 4 shows that the optimal demand does not increase as the weakening factor increases, when  $Bb \in \left[\frac{1}{2}, 1\right]$ , the optimal demand decreases as the weakening factor decreases; and in other situations, the optimal demand increases as the weakening factor decreases. Generally, we consider that if the weakening factor increases, which means that online advertising has less effect on traditional advertising, the optimal demand will increase. The weakening factor does not always harm the advertising strategies, because the complementary effect is greater than the substitution effect. This conclusion reflects that the unit online advertising cost  $B$  and price elasticity  $b$  can influence the customer's choice and optimal demand. On the other hand, Corollary 4 also explains the connection between the weakening factor and optimal price and revenue. When sellers take combination advertising, a larger weakening factor represents combination advertising is more effective, sellers can earn more profit with combi-

nation advertising. At the same time, sellers can charge a higher optimal price due to diminishing marginal utility.

### 2.3 Advertising choice

We analyze the separate advertising strategies of traditional advertising and combination advertising. In particular, sellers decide which mode of advertising should be adopted. Sellers always want to obtain the maximum profit. Based on this, we compare the maximum profit with two different advertising modes.

**Proposition 3** Without advertising budget constraint, sellers adopt combination advertising when the weakening factor  $\lambda$  satisfies  $\sqrt{1 - \frac{1}{2Bb}} - \left(1 - \sqrt{1 - \frac{1}{2Bb}}\right) \frac{a}{m} < \lambda < 1$ .

For sellers, the weakening factor is the main factor that affects advertising strategies. In our minds, if online advertising has a negative effect on traditional advertising, sellers will suffer the loss from combination advertising, that is, sellers prefer traditional advertising to combination advertising. From Proposition 3, to sellers, the weakening factor satisfying Proposition 3 indicates that the online advertising and traditional advertising are more complementary, and sellers can capture more customers with combination advertising. That is, the product's unit advertising cost is reduced or advertising's effectiveness is higher, so that sellers can gather more revenue. The cost of online advertising may affect sellers' advertising strategies. Different from traditional advertising, online advertising's cost is affected by the unit cost and the scope of delivery. So, we have the following corollary.

**Corollary 5** Without advertising budget constraint, the unit advertising cost  $B \leq \frac{(a + m)^2}{2bm(1 - \lambda)[(1 + \lambda)m + 2a]}$ , sellers adopt combination advertising. In some certain scenarios, combination advertising is not a Pareto optimum relative to traditional advertising.

If sellers want to adopt combination advertising based on Proposition 3, the unit online advertising cost should satisfy Corollary 5. In past research, the unit online advertising cost is not considered, but it is as important as the unit traditional advertising cost. If the unit online advertising cost is too high  $\left(\geq \frac{(a + m)^2}{2bm(1 - \lambda)[(1 + \lambda)m + 2a]}\right)$ , and the delivery scope is not large, sellers suffer the loss from online advertising. We compare the optimal demand, price and revenue of sellers adapting different advertising modes. We demonstrate that in some certain scenarios, the combination advertising is not Pareto optimum to traditional advertising.

When  $\lambda \in \left[\left(\sqrt{1 - \frac{1}{2Bb}} - 1\right) \frac{a}{m} + \sqrt{1 - \frac{1}{2Bb}}, 1\right]$ , the optimal price, maximum revenue and optimal demand are higher than those of the traditional advertising; but  $\lambda \in \left[1 - \frac{1}{2Bb} - \frac{a}{2Bbm}, \left(\sqrt{1 - \frac{1}{2Bb}} - 1\right) \frac{a}{m} + \sqrt{1 - \frac{1}{2Bb}}\right]$ , the

revenue of combination advertising is less than that of traditional advertising, but the optimal price and demand are greater than that of traditional advertising. So, sellers should consider the optimal revenue and the optimal potential demand simultaneously.

### 3 Analysis of Advertising Strategies with Budget

Consider that sellers' budgets are constrained and smaller than the optimal advertising cost  $\frac{B}{2} \left( \frac{a + \lambda m}{2Bb - 1} \right)^2 + A$ .

How does the advertising budget affect different advertising mode strategies? How do sellers choose advertising strategies? How will they allocate their advertising budget with combination advertising? In this section, we analyze the advertising and pricing strategies with a budget constraint  $M \left( M < M_{\max} = \frac{B}{2} \left( \frac{a + \lambda m}{2Bb - 1} \right)^2 + A \right)$ .

#### 3.1 Traditional advertising strategies

Following the traditional advertising model, by investing  $M$  in traditional advertising, sellers can obtain an additional demand  $\frac{M}{A}m$  from traditional advertising. We can derive sellers' traditional advertising strategies and pricing strategies.

**Proposition 4** Sellers invest in traditional advertising if the traditional advertising conversion rate satisfies  $\frac{m}{A} >$

$\frac{4b}{2a + \frac{M}{A}m}$ . Sellers adopt traditional advertising with a

budget constraint. The optimal price is  $\frac{a}{2b} + \frac{Mm}{2bA}$ , and the maximum profit is  $\frac{1}{4b} \left( a + \frac{M}{A}m \right)^2 - M$ .

Proposition 4 represents sellers' optimal advertising and pricing strategies with budget constraint. The basic market and traditional advertising conversion rate affect advertising and pricing strategies which is similar to traditional advertising strategies without a budget constraint. However, the budget constraint influences sellers' budget allocations and price strategies differently. From Proposition 4, we derive the following corollary.

**Corollary 6** A larger advertising budget may lead to a higher price, but it cannot guarantee an increased profit.

When sellers invest more in advertising, they need to transfer the advertising cost to customers, so increasing investment increases the price. Interestingly, sellers are not guaranteed to obtain more profit with a greater budget. Sellers' profits decrease first and increase with an increasing budget.

#### 3.2 Combination advertising strategies

Consider that the sellers' advertising budget is  $M$ , and

they invest  $kM$  in online advertising, so that the budget of traditional advertising is  $(1 - k)M$ . We assume that the advertising covering scope is linear with the investment, and the increment of traditional advertising is  $\frac{(1 - k)Mm}{A}$ .

We can derive that sellers' revenue is given by  $\pi = p \left( a - bp + \frac{\lambda(1 - k)Mm}{A} + \sqrt{\frac{2kM}{B}} \right) - M$ . Using a standard analysis, we can obtain the optimal strategies of sellers.

**Proposition 5** With an advertising budget constraint, the optimal ratio of investing to online advertising is  $k^* = \frac{A^2}{2MBm^2\lambda^2}$ . The optimal delivery scope of online advertising is  $t^* = \frac{A}{\lambda Bm}$ , and the traditional advertising delivery

scope is  $\frac{\lambda Mm}{A} - \frac{A}{2\lambda Bm}$ . The optimal price is  $p^* = \frac{a}{2b} +$

$\frac{\lambda m}{2b} \frac{M}{A} + \frac{1}{4\lambda Bbm}$ , and sellers' maximum revenue is  $\pi^* = \frac{1}{4b} \left( a + \frac{\lambda Mm}{A} + \frac{1}{2\lambda Bm} \right)^2 - M$ .

The optimal strategies of sellers guarantee the sellers' payoffs with a budget constraint. From Proposition 5, we can see that the optimal online advertising delivery scope is not connected to the budget  $M$ , but online advertising and the budget constraint affect sellers' revenue and demands.

**Corollary 7** With the budget constraint, the maximum revenue is nonlinearly related to the budget.

Sellers' maximum revenue is affected by the budget constrain nonlinearly. We find that when the budget constrain satisfies  $M^* = \frac{A^2}{2\lambda^2 m^2} - \frac{A}{\lambda am} - \frac{A}{2B\lambda^2 m^2}$ , sellers can obtain the maximum profit  $\frac{1}{4b} \left( a + \frac{A}{2\lambda m} - \frac{1}{a} \right)^2 - \frac{A^2}{2\lambda^2 m^2} -$

$\frac{A}{\lambda am} - \frac{A}{2B\lambda^2 m^2}$ . Therefore, when the budget is constrained, which is greater than  $M^*$  but less than  $M_{\max}$ , sellers choose  $M^*$  to set the advertising strategies. When the budget  $M < M^*$ , sellers should follow the strategies of Proposition 4. Corollary 7 does not follow common sense. Usually, with a budget constraint, sellers should invest as much as possible to follow the optimal strategies more closely. From Corollary 7, we find that if the budget is constrained, sellers do not need to invest as much as possible.

**Corollary 8** With the budget constraint fixed, the optimal demand is nonlinearly related to the weakening factor. Sellers can guarantee that their optimal revenue will be more than  $\frac{1}{4b} \left( a + \sqrt{\frac{2M}{AB}} \right)^2 - M$ .

From the analysis of section 2.2, we demonstrate that the optimal demand is nonlinear with the weakening fac-

tor. In Corollary 8, we find that with budget constraints, the optimal demand is also nonlinear with the weakening factor. When  $\lambda \leq \frac{1}{m} \sqrt{\frac{A}{2BM}}$ , the optimal demand decreases when  $\lambda$  decreases; and when  $\lambda > \frac{1}{m} \sqrt{\frac{A}{2BM}}$ , the optimal demand increases with the increase in  $\lambda$ . When  $\lambda = \frac{1}{m} \sqrt{\frac{A}{2BM}}$ , the optimal demand is  $D_{\min}^* = \frac{a}{2} + \sqrt{\frac{M}{2AB}}$ , and we can guarantee that the minimum optimal revenue is  $\frac{1}{4b} \left( a + \sqrt{\frac{2M}{AB}} \right)^2 - M$ . From these analyses, we can see that in any scenarios, sellers can obtain more than  $\frac{1}{4b} \left( a + \sqrt{\frac{2M}{AB}} \right)^2 - M$ . That is, if the optimal revenue is less than  $\frac{1}{4b} \left( a + \sqrt{\frac{2M}{AB}} \right)^2 - M$ , sellers should abandon combination advertising with a budget constraint.

### 3.3 Advertising choice

We analyze which advertising mode sellers will choose with an advertising budget constraint. Sellers chase more profit with advertising by comparing the profits of different advertising modes.

**Proposition 6** 1) When advertising budget  $M \in [A, A + \frac{B}{2} \left( \frac{a + \lambda m}{2Bb - 1} \right)^2]$ , meanwhile, the weakening factor  $\lambda \in \left[ 0, \frac{1}{m} \left( \frac{L}{2M} - \sqrt{\frac{L^2}{4M^2} - \frac{A}{2BM}} \right) \cup \left[ \frac{1}{m} \left( \frac{L}{2M} + \sqrt{\frac{L^2}{4M^2} - \frac{A}{2BM}} \right), 1 \right] \right]$ , where  $L = \sqrt{4b + (M - A) + m^2 + a^2} - a$ , sellers prefer combination advertising with the maximum revenue equal to  $\frac{1}{4b} \left( a + \frac{\lambda M m}{A} + \frac{1}{2\lambda B m} \right)^2 - M$ . Otherwise, sellers adopt traditional advertising.

2) When advertising budget  $M \in [0, A]$ , and the weakening factor satisfies  $\lambda \in \left[ 0, 1 - \sqrt{1 - \frac{A}{2BMm^2}} \right]$ , sellers prefer combination advertising with the maximum revenue equal to  $\frac{\left( a + \frac{M}{A} m \right)^2}{4b} - M$ . Otherwise, sellers adopt traditional advertising.

From Proposition 6, we can see that when sellers' budgets cannot cover the whole market, they should consider the budget and the weakening factors simultaneously when they choose the advertising mode. When the weakening factor is less than  $\frac{1}{m} \left( \frac{L}{2M} - \sqrt{\frac{L^2}{4M^2} - \frac{A}{2BM}} \right)$ , sellers adopt combination advertising. In this paper, a smaller  $\lambda$  indicates that online advertising has a strong weakening effect on traditional advertising, which means that sellers prefer online advertising to traditional advertising due to

the higher effectiveness of online advertising. Nowadays, online advertising is becoming more and more popular and sellers can obtain the potential customers' personal information by other means, but due to traditional advertising's complementary, traditional advertising still plays an important role in the sellers' advertising strategies.

From Proposition 6, we can derive the following corollary.

**Corollary 9** Combination advertising is not a Pareto optimum relative to traditional advertising when sellers adopt combination advertising. When the budget  $M \in \left[ 0, \frac{A}{2\lambda(1-\lambda)Bm^2} \right]$ , sellers prefer combination advertising.

Corollary 9 shows that in some certain scenarios, with an advertising budget constraint, sellers can obtain more revenue with less demand due to the greater effectiveness of the advertising and the complementary effect. In Proposition 3, we discuss the weakening effect on the advertising strategies. Corollary 9 also shows how to choose advertising strategies when the budget is less than  $A$ . When sellers are constrained with less budget, they would like to choose a higher effective advertising mode. Nevertheless, if the budget is greater than  $\frac{A}{2\lambda(1-\lambda)Bm^2}$ , sellers should compare the revenues of combination advertising and traditional advertising.

## 4 Conclusion

One of the main challenges that sellers face in advertising and media planning is how they choose an advertising mode with budget constraint. Meanwhile, if the budget cannot cover the market, the decision makers should decide how to allocate the budget. We find that besides the budget, the weakening factor is another main factor that influences sellers' advertising strategy. Different from the previous research on online advertising and traditional advertising, sellers should consider the complementary relationship between the two advertising modes and adopt the combination of online advertising and traditional advertising (combination advertising). Traditional advertising still plays an important role in media. In some certain scenarios, traditional advertising is better than combination advertising. Based on these, we study the pricing and advertising strategies with/without a budget constraint and give the optimal strategies. Furthermore, we find that, even if the sellers choose combination advertising, the output of combination advertising is not a Pareto optimum relative to traditional advertising no matter whether with/without a budget constraint. These conclusions have a good reference and guiding significance for corporate decision makers to set advertising strategies and design advertising budgets.

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## 预算限制下的组合广告

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**摘要:**采用博弈论和优化模型分析了企业广告模式选择策略及面对广告预算限制时如何制定广告策略和定价策略. 研究表明, 广告预算和网络广告对传统广告的削弱会影响企业的广告、定价决策及预算分配. 一定范围内组合广告的效果优于传统广告. 而网络广告对传统广告的削弱作用将会对企业的广告策略和定价策略产生非线性影响. 此外, 预算限制条件下企业选择组合广告后, 商家可以制定最优的广告预算分配比例; 一定条件下, 商家能够以低于预算的广告支出获得更多的利益. 商家采用组合广告并且预算足够时, 不会无限地投递传统广告. 此外, 与预算充足情况相比, 商家面临广告预算限制时, 不会减少网络广告的预算分配, 因为网络广告具有更高的效率.

**关键词:**组合广告; 预算分配; 传统广告; 网络广告; 广告预算

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