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BiHom-H-pseudoalgebras and their constructions

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Abstract: The definition and an example of BiHom-associative H-pseudoalgebra are given. A BiHom-H-pseudoalgebra is an H-pseudoalgebra(A, μ) with two maps $\alpha, \beta \in \mathrm{Hom}_H(A, A)$ satisfying the BiHom-associative law which generalizes BiHom-associative algebras and associative H-pseudoalgebras. Secondly, a method which is called the Yau twist of constructing BiHom-associative H-pseudoaglebra $(A, (I_{H\otimes H} \otimes_H \alpha)\mu, \alpha, \beta)$ from an associative H-pseudoalgebra (A, μ) and two maps of H-pseudoalgebras α, β, is introduced. Thirdly, a generalized form of the Yau twist is discussed. It concerns constructing a BiHom-associative H-pseudoalgebra $(A, \mu(\alpha\otimes\beta), \alpha^{\sim}\alpha, \beta^{\sim}\beta)$ from a BiHom-associative H-pseudoalgebra $(A, \mu, \alpha^{\sim}, \beta^{\sim})$ and two maps $\alpha, \beta \in \mathrm{Hom}_H(A, A)$. Finally, a method of constructing BiHom-associative H-pseudoalgebra on tensor product space $A \otimes B$ of two BiHom-associative H-pseudoalgebras is given.

Key words: BiHom-associative H-pseudoalgebrsa; Yau twist; tensor product BiHom-associative H-pseudoalgebras

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BiHom-algebra was introduced by Graziani et al[1]. The concept arose from the research of algebra in group-Hom categories, and it is an important generalization of Hom-algebra[2-4]. H-pseudoalgebra was introduced by Bakalov et al[5]. It is an algebra in the pseudotensor category $M^*(H)$, and it is a generalization of the conformal algebra introduced by Kac[6]. We can also regard it as multidimensional conformal algebra. The structure of H-pseudoalgebra has some connections with mathematical physics and nonlinear equations[7-9] and it has been developed into more general forms[10-11].

The purpose of this paper is to define an algebraic system which generalizes BiHom-algebra and H-pseudoalgebra, so as to research its basic properties.

1 Preliminaries

Remark 1 Throughout this paper, we define that

1) k is a field, and the base vector spaces of all algebraic structures are over k.

2) H is a cocommutative Hopf algebra.

3) We use the sweedler notation to express the coproduct of H: $\Delta(h) = h_1 \otimes h_2$, for any $h \in H$.

Definition 1[5] An associative H-pseudoalgebra is a pair $(A, \mu = *)$ which satisfies:

1) A is an H-module.

2) $u \in \mathrm{Hom}_{H\otimes H}(A\otimes A, (H\otimes H)\otimes_H A)$ and we denote $\mu(a\otimes b) = a * b$. It is equivalent to (H-bilinearity) $fa * gb = ((f\otimes g)\otimes_H 1)(a * b)$.

3) (associative law) $(a * b) * c = a * (b * c)$.

Remark 2 We explicitly describe the associative law of H-pseudoalgebra.

If we denote

$$a * b = \sum_i (f_i \otimes g_i) \otimes_H e_i,\ e_i * c = \sum_{i,j} (f_{ij} \otimes g_{ij}) \otimes_H e_{ij}$$

$$b * c = \sum_i (h_i \otimes l_i) \otimes_H d_i,\ a * d_i = \sum_{i,j} (h_{ij} \otimes l_{ij}) \otimes_H d_{ij}$$

Then,

$$(a * b) * c = \sum_{i,j} (f_i f_{ij_1} \otimes g_i f_{ij_2} \otimes g_{ij}) \otimes_H e_i$$

$$a * (b * c) = \sum_{i,j} (h_{ij} \otimes h_i l_{ij_1} \otimes l_i l_{ij_2}) \otimes_H d_{ij}$$

Definition 2[5] A map of H-pseudoalgebras from (A, μ_A) to (B, μ_B) is defined as follows:

1) $f \in \mathrm{Hom}_H(A, B)$;

2) $\mu_B(f\otimes f) = (I_{H\otimes H}\otimes f)\mu_A$.

Definition 3[10] A Hom-associative H-pseudoalgebra is a triple (A, μ, α) which satisfies: 1) (A, μ) is an H-pseudoalgebra; 2) $\alpha \in \mathrm{Hom}_H(A, A)$; 3) $(a * b) * \alpha(c) = \alpha(a) * (b * c)$.

Definition 4[1] A multiplicative BiHom-associative algebra is a triple (A, α, β), which satisfies:

1) A is an algebra;

2) $\alpha, \beta \in \mathrm{Hom}_k(A, B)$;

3) $(a * b) * \beta(c) = \alpha(a) * (b * c)$;

4) $\alpha(ab) = \alpha(a)\alpha(b)$, $\beta(ab) = \beta(a)\beta(b)$;

5) $\alpha\beta = \beta\alpha$.

2 BiHom-Associative H-Pseudoalgebras

Definiton 5 A BiHom-associative H-pseudoalgebra is a quadruple $(A, \mu = *, \alpha, \beta)$ which satisfies:

1) A is an H-module.

2) $u \in \mathrm{Hom}_{H\otimes H}(A\otimes A, (H\otimes H)\otimes_H A)$ and we denote

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$u(a\otimes b) = a * b$;

3) $\alpha, \beta \in \mathrm{Hom}_H(A, A)$;

4) $\alpha\beta = \beta\alpha$;

5) (BiHom-associative law) $\alpha(x) * (y * z) = (x * y) * \beta(z)$.

Remark 3 A is a BiHom-associative H-pseudoalgebra.

1) If $H = k$, A is a BiHom-associative algebra.

2) If $\alpha = \beta = I$, A is an associative H-pseudoalgebra.

3) If $\alpha = \beta$, A is a Hom-associative H-pseudoalgebra.

Example 1 (A, α, β) is a finite dimensional BiHom-associative H-pseudoalgebra, H is a Hopf algebra. Then, $(H\otimes A, *, I_H\otimes\alpha, I_H\otimes\beta)$ is a BiHom-associative algebra by

$$(f\otimes a) * (g * b) = (f\otimes g)\otimes_H(1\otimes ab)$$

Definiton 6 A BiHom-associative H-pseudoalgebra (A, μ, α, β) is multiplicative if

$$(I_{H\otimes H}\otimes_H\alpha)\mu = \mu(\alpha\otimes\alpha),\ (I_{H\otimes H}\otimes_H\beta)\mu = \mu(\beta\otimes\beta)$$

Example 2 We take $\alpha = \beta = I$, the BiHom-assciative H-pseudoalgebra is multiplicative.

Definition 7 Let $(A, \mu_B, \alpha_B, \beta_B)$, $(B, \mu_B, \alpha_B, \beta_B)$ be two (multiplicative) BiHom-associative H-pseudoalgebras, and a map of (multiplicative) BiHom-associative H-pseudoalgebras is defined as

1) $f \in \mathrm{Hom}_H(A, B)$;

2) $(I_{H\otimes H}\otimes_H f)\mu_A = \mu_B(f\otimes f)$;

3) $\alpha_B f = f\alpha_A, \beta_B f = f\beta_A$.

Now, we introduce a method of constructing BiHom-associative H-pseudoalgebras from associative H-pseudoalgebras.

Theorem 1 1) (A, μ_A) is an associative H-pseudoalgebra, $\alpha_A: A\to A$, $\beta_A: A\to A$ are two maps of H-pseudoalgebra which satisfies

$$\alpha_A\circ\beta_A = \beta_A\circ\alpha_A$$
$$(I_{H\otimes H}\otimes_H\alpha_A)\mu_A = \mu_A(\alpha_A\otimes\beta_A)$$

Then, $(A, \mu^* = (I_{H\otimes H}\otimes_H\alpha_A)\mu_A, \alpha_A, \beta_A)$ is a multiplicative BiHom-associative H-pseudoalgebra.

2) (B, μ_B) is an H-pseudoalgebra, and $\alpha_B, \beta_B: B\to B$ are maps of H-pseudoalgebras which satisfies

$$\alpha_B\circ\beta_B = \beta_B\circ\alpha_B$$
$$(I_{H\otimes H}\otimes_H\alpha_B)\mu_B = \mu_B(\alpha_B\otimes\beta_B)$$

$f: A\to B$ is a map of H-pseudoalgebras which satisfies

$$f\alpha_A = \alpha_B f,\ f\beta_A = \beta_B f$$

Then, $f: (A, (I_{H\otimes H}\otimes_H\alpha_A)\mu_A, \alpha_A, \beta_A)\to(B, (I_{H\otimes H}\otimes_H\alpha_B)\mu_B, \alpha_B, \beta_B)$ is a map of multiplicative BiHom-associative H-pseudoalgebras.

Proof We denote $\mu^*(a\otimes b) = a *^* b$. For any $f, g \in H$, $a, b\in A$,

$$\begin{aligned}(fa *^* gb) &= (I_{H\otimes H}\otimes_H\alpha_A)\mu_A(fa\otimes gb) = \\ &(I_{H\otimes H}\otimes_H\alpha)((f\otimes g)\otimes_H 1)(a * b) = \\ &((f\otimes g)\otimes_H 1)(I_{H\otimes H}\otimes_H\alpha_A)(a * b) = \\ &((f\otimes g)\otimes_H 1)(a *^* b)\end{aligned}$$

Then μ^* is a pseudoproduct for A.

The proof of BiHom H-associative law is as follows:

$$\begin{aligned}a *^* b &= \alpha_A(a) * \beta_A(b) = (I_{H\otimes H}\otimes_H\alpha_A)(a * b) = \\ &\sum_i (f_i\otimes g_i)\otimes_H\alpha_A(e_i)\end{aligned}$$

$$\begin{aligned}\alpha_A(e_i) *^* \beta_A(c) &= (I_{H\otimes H}\otimes_H\alpha_A)(\alpha_A(e_i) * \beta_A(c)) = \\ &(I_{H\otimes H}\otimes_H\alpha_A)(I_{H\otimes H}\otimes_H\alpha_A)(e_i\otimes c) = \\ &\sum_{i,j} f_{ij}\otimes g_{ij}\otimes_H{\alpha_A}^2(e_{ij})\end{aligned}$$

So,

$$\begin{aligned}&(a *^* b) *^* \beta_A(c) = \\ &\sum_{i,j}(f_i f_{ij_1}\otimes g_i f_{ij_2}\otimes g_{ij})\otimes_H{\alpha_A}^2(e_{ij}) = \\ &(I_{H\otimes H}\otimes_H{\alpha_A}^2)((a * b) * c) = \\ &(I_{H\otimes H}\otimes_H{\alpha_A}^2)(a * (b * c)) = (I_{H\otimes H}\otimes_H{\alpha_A}^2)\cdot \\ &(\sum_{i,j}(h_{ij}\otimes h_i l_{ji_1}\otimes l_i l_{ij_2}))\otimes_H d_{ij})\end{aligned}$$

Similarly,

$$\begin{aligned}&\alpha_A(a) *^* (b *^* c) = (I_{H\otimes H}\otimes_H{\alpha_A}^2)\cdot \\ &(\sum_{i,j}(h_{ij}\otimes h_i l_{ij_1}\otimes l_i l_{ij_2})\otimes_H d_{ij}) = \\ &(a *^* b) *^* \beta_A(c)\end{aligned}$$

The proof of multiplicative for $(A, \mu^* = (I_{H\otimes H}\otimes_H\alpha)\mu, \alpha, \beta)$ is

$$\begin{aligned}\alpha(a) *^* \alpha(b) &= (I_{H\otimes H}\otimes_H\alpha)(a *^* b), \beta(a) *^* \beta(b) = \\ &(I_{H\otimes H}\otimes_H\alpha)(\beta(a) * \beta(b)) = \\ &(I_{H\otimes H}\otimes_H\alpha)(I_{H\otimes H}\otimes_H\beta)(a * b) = \\ &(I_{H\otimes H}\otimes_H\beta)(I_{H\otimes H}\otimes_H\alpha)(a * b) = \\ &(I_{H\otimes H}\otimes_H\beta)(a *^* b)\end{aligned}$$

$$\begin{aligned}(I_{H\otimes H}\otimes_H f)&(I_{H\otimes H}\otimes_H\alpha_A)\mu_A(a\otimes b) = \\ &(I_{H\otimes H}\otimes_H f)(\alpha_A(a) * \beta_A(b)) = \\ &f\alpha_A(a) * f\beta_A(b) = \\ &\alpha_B f(a) * \beta_B f(b) = \\ &(I_{H\otimes H}\otimes_H\alpha_B)(f(a) * f(b)) = \\ &(I_{H\otimes H}\otimes_H\alpha_B)\mu_B(f\otimes f)(a\otimes b)\end{aligned}$$

Example 2 H is a cocommutative Hopf algebra and $G(H)$ is the set of group like elements of H. $H\{e_1, e_2\}$ is a free H-pseudoalgebra of rank 2, with pseudoproduct given by

$$e_1 * e_1 = a\otimes_H e_1,\ e_2 * e_2 = b\otimes_H e_2$$
$$e_1 * e_2 = a\otimes_H e_2,\ e_2 * e_1 = -a\otimes_H e_2$$

We define $\alpha, \beta: H\{e_1, e_2\}\to H\{e_1, e_2\}$:

$$\alpha(e_1) = ge_1,\ \alpha(e_2) = ge_2$$
$$\beta(e_1) = he_1,\ \beta(e_2) = he_2,\ g, h\in G(H)$$

satisfying

$$(g\otimes g)a=(g\otimes h)a,\ (g\otimes g)b=(g\otimes h)$$

Then, α, β are endomorphisms of $H\{e_1, e_2\}$, and $(H\{e_1,e_2\},\mu^1=(I_{H\otimes H}\otimes\alpha)\mu,\alpha,\beta)$ is a multiplicative BiHom-associative H-pseudoalagebra with pseudoproduct μ^1 given by

$$\mu^1(e_1\otimes e_1)=(g\otimes g)a\otimes_H e_1$$
$$\mu^1(e_2\otimes e_2)=(g\otimes g)b\otimes_H e_2$$
$$\mu^1(e_1\otimes e_2)=(g\otimes g)a\otimes_H e_2$$
$$\mu^1(e_2\otimes e_1)=-(g\otimes g)a\otimes_H e_2$$

Definition 8 The BiHom-associative H-pseudoalgebra $(A,\mu^*=(I_{H\otimes H}\otimes_H\alpha)\mu,\alpha,\beta)$ is called the Yau twist of H-pseudoalgebra(A,μ) and is denoted by $A_{\alpha,\beta}$.

The following is a generalization of Theorem 1.

Theorem 2 $(A,\mu,\alpha^{\sim},\beta^{\sim})$ is a multiplicative BiHom-associative H-pseudoalgebra, and $\alpha,\beta\in \mathrm{Hom}_H(A,A)$ satisfies

$$(I_{H\otimes H}\otimes\alpha)\mu=\mu(\alpha\otimes\beta),(I_{H\otimes H}\otimes\alpha)\mu=\mu(\alpha\otimes\alpha)$$
$$(I_{H\otimes H}\otimes\beta)\mu=\mu(\beta\otimes\beta)$$

and each of α, β, $\alpha^{\sim}$, $\beta^{\sim}$ can commute with others.

Then, $(A,\mu(\alpha\otimes\beta),\ \alpha^{\sim}\circ\alpha,\beta^{\sim}\circ\beta)$ is a multiplicative BiHom-associative H-pseudoalgebra.

Proof We denote $\mu(\alpha\otimes\beta)(a\otimes b)\equiv a*^{*}b$.

The proof of BiHom-associative law is

$$\begin{aligned}&(b*^{*}c)=(I_{H\otimes H}\otimes_H\alpha)(b*c)=\\&\quad(I_{H\otimes H}\otimes_H\alpha)(\sum_i(h_i\otimes l_i)\otimes d_i)=\\&\quad\sum_i(h_i\otimes l_i)\otimes\alpha(d_i)\\&\alpha^{\sim}\alpha(a)*^{*}\alpha(d_i)=\alpha\alpha^{\sim}(a)*^{*}\alpha(d_i)=\\&\quad(I_{H\otimes H}\otimes_H\alpha)(\alpha\alpha^{\sim}(a)*\alpha(d_i))=\\&\quad(I_{H\otimes H}\otimes_H\alpha^2)(\alpha^{\sim}(a)*d_i)\end{aligned}$$

So,

$$\alpha^{\sim}\alpha(a)*^{*}(b*^{*}c)=(I_{H\otimes H\otimes H}\otimes_H\alpha^2)(\alpha^{\sim}(a)*(b*c))$$

Similarly,

$$\begin{aligned}&(a*^{*}b)*^{*}\beta^{\sim}\beta(c)=\\&\quad(I_{H\otimes H\otimes H}\otimes_H\alpha^2)(a*b)*\beta^{\sim}(c)\end{aligned}$$

The proof of $\alpha^{\sim}\alpha(a)*^{*}\alpha^{\sim}\alpha(b)=\beta^{\sim}\beta(a)*^{*}\beta^{\sim}\beta(b)$ is

$$\begin{aligned}&\alpha^{\sim}\alpha(a)*^{*}\alpha^{\sim}\alpha(b)=\\&\quad(I_{H\otimes H}\otimes_H\alpha)(\alpha^{\sim}\alpha(a)*\alpha^{\sim}\alpha(b))=\\&\quad(I_{H\otimes H}\otimes_H\alpha)(I_{H\otimes H}\otimes_H\alpha^{\sim})(\alpha(a)*\alpha(b))=\\&\quad(I_{H\otimes H}\otimes_H\alpha\alpha^{\sim})(a*^{*}b)\end{aligned}$$

Similarly,

$$\beta^{\sim}\beta(a)*^{*}\beta^{\sim}\beta(b)=(I_{H\otimes H}\otimes_H\beta^{\sim}\beta)(a*^{*}b)$$

Theorem 3 $(A,\mu_A,\alpha_A,\beta_A)$ is a multiplicative BiHom-associative H_1-pseudoalgebra and $(B,\mu_B,\alpha_B,\beta_B)$ is a multiplicative BiHom-associative H_2-pseudoalgebra, and then$(A\otimes B,\mu_{A\otimes B},\alpha_A\otimes\alpha_B,\beta_A\otimes\beta_B)$ is a multiplicative BiHom-associative $H=H_1\otimes H_2$-pseudoalgebra by

$$\begin{aligned}&\mu_{A\otimes B}((a_1\otimes b_1)\otimes(a_2\otimes b_2))=\\&\quad\sum_i(f_i\otimes F_I\otimes g_i\otimes G_I)\otimes_H(e_i\otimes E_I)\end{aligned}$$

We denote

$$a_1*a_2=\sum_i(f_i\otimes g_i)\otimes e_i,b_1*b_2=\sum_I(h_I\otimes l_I)\otimes_H d_I$$

Proof The $H=H_1\otimes H_2$-module structure of $A\otimes B$ is defined as $(g\otimes h)(a\otimes b)=ga\otimes hb$ and the pseudoproduct of $A\otimes B$ is well defined. The H-bilinearity of $\mu_{A\otimes B}$ is obvious. We define some symbols:

$$\begin{aligned}a_2*a_3&=\sum_i(h_i\otimes l_i)\otimes_H d_i,\ \alpha_A(a_1)*d_i=\\&\quad\sum_{i,j}(h_{ij}\otimes l_{ij})\otimes_H d_{ij}\\a_1*a_2&=\sum_i(f_i\otimes g_i)\otimes_H e_i,\ e_i*\beta_A(d_i)=\\&\quad\sum_{i,j}(f_{ij}\otimes g_{ij})\otimes_H e_{ij}\\b_2*b_3&=\sum_I(H_I\otimes L_I)\otimes_H D_I,\ \alpha_B(b_1)*D_i=\\&\quad\sum_{I,J}(H_{IJ}\otimes L_{IJ})\otimes_H D_{IJ}\\b_1*b_2&=\sum_I(F_I\otimes G_I)\otimes_H E_I,\ E_I*\beta_A(D_I)=\\&\quad\sum_{I,J}(F_{IJ}\otimes G_{IJ})\otimes_H E_{IJ}\end{aligned}$$

The proof of BiHom-associative law is as follows:

$$\begin{aligned}&(a_2\otimes b_2)*(a_3\otimes b_3)=\\&\quad\sum_{iI}(h_i\otimes H_I\otimes l_i\otimes L_I)\otimes_H(d_i\otimes D_I)\cdot\\&(\alpha_A(a_1)\otimes\alpha_B(b_2))*(d_i\otimes D_I)=\\&\quad\sum_{ij,IJ}(h_{ij}\otimes H_{IJ}\otimes l_{ij}\otimes L_{IJ})\otimes_H(d_{ij}\otimes D_{IJ})\end{aligned}$$

So,

$$\begin{aligned}&(\alpha_A\otimes\alpha_B)(a_1\otimes b_1)*((a_2\otimes b_2)*(a_3\otimes b_3))=\\&\quad\sum_{ij,IJ}(h_{ij}\otimes H_{IJ})\otimes(h_i\otimes H_I)(l_{ij_1}\otimes L_{IJ_1})\otimes\\&\quad(l_i\otimes L_I)(l_{ij_2}\otimes L_{IJ_2})\otimes_H(d_{ij}\otimes D_{IJ})\end{aligned}$$

Similarly,

$$\begin{aligned}&((a_1\otimes b_1)\otimes(a_2\otimes b_2))*(\beta(a_3)\otimes\beta(b_3))=\\&\quad\sum_{ij,IJ}(f_i\otimes F_I)(f_{ij_1}\otimes F_{IJ_1})\otimes(g_i\otimes G_I)(f_{ij_2}\otimes F_{IJ_2})\otimes\\&\quad(g_{ij}\otimes G_{IJ})\otimes_H(e_{ij}\otimes E_{IJ})\end{aligned}$$

By the BiHom-associative law of $(A,\mu_A,\alpha_A,\beta_A)$ and $(B,\mu_B,\alpha_B,\beta_B)$,

$$\begin{aligned}&(\alpha_A\otimes\alpha_B)(a_1\otimes b_1)*((a_2\otimes b_2)*(a_3\otimes b_3))=\\&\quad((a_1\otimes b_1)\otimes(a_2\otimes b_2))*(\beta_A(a_3)\otimes\beta_B(b_3))\end{aligned}$$

The proof of the multiplicative law is

$$(I_{H_1}\otimes I_{H_2}\otimes I_{H_1}\otimes I_{H_2}\otimes \alpha_A\otimes \alpha_B)((a_1\otimes b_1)*(a_2\otimes b_2)) = (I_{H_1}\otimes I_{H_2}\otimes I_{H_1}\otimes I_{H_2}\otimes \alpha_A\otimes \alpha_B)\cdot\left(\sum_{iI}(f_i\otimes F_I\otimes g_i\otimes G_I)\otimes_H(e_i\otimes E_I)\right) = \sum_{iI}(f_i\otimes F_I\otimes g_i\otimes G_I)\otimes_H(\alpha_A(e_i)\otimes\alpha_B(E_I))$$

On the other side,

$$\alpha_A(a_1)*\alpha_A(a_2) = (I_{H_1}\otimes I_{H_1}\otimes\alpha_A)(a_1\otimes a_2) = \sum_i(f_i\otimes g_i)\otimes_{H_1}\alpha_A(e_i)\quad \alpha_B(b_1)*\alpha_B(b_2) = (I_{H_2}\otimes I_{H_2}\otimes\alpha_B)(b_1\otimes b_2) = \sum_I(F_I\otimes G_I)\otimes_{H_2}\alpha_B(E_I)$$

Therefore,

$$(I_{H_1}\otimes I_{H_2}\otimes I_{H_1}\otimes I_{H_2}\otimes \alpha_A\otimes \alpha_B)((a_1\otimes b_1)*(a_2\otimes b_2)) = (\alpha_A\otimes\alpha_A)(a_1\otimes b_1)*(\alpha_B\otimes\alpha_B)(a_2\otimes b_2)$$

Similarly,

$$(I_{H_1}\otimes I_{H_2}\otimes I_{H_1}\otimes I_{H_2}\otimes\beta_A\otimes\beta_B)((a_1\otimes b_1)*(a_2\otimes b_2)) = (\beta_A\otimes\beta_B)(a_1\otimes b_1)*(\alpha_B\otimes\alpha_B)(a_2\otimes b_2)$$

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BiHom-*H*-伪代数及其构造

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摘要：首先，给出了 BiHom-结合 H-伪代数的定义与例子，一个 BiHom-结合 H-伪代数由一个 H-伪代数(A,μ)和满足 BiHom-结合律的 2 个映射 $\alpha,\beta\in \mathrm{Hom}_H(A,A)$ 构成，其为 BiHom-结合代数和结合 H-伪代数的推广. 然且，介绍了名为 Yau 扭曲的方法，该方法是由一个结合 H-伪代数(A,μ)和 2 个 H-伪代数同态 α,β 构造 BiHom-结合 H-伪代数$(A,(I_{H\otimes H}\otimes_H\alpha)\mu,\alpha,\beta)$. 同时，介绍了 Yau 扭曲的推广形式，即由一个 BiHom-结合 H-伪代数$(A,\mu,\alpha^{\sim},\beta^{\sim})$和 2 个映射 $\alpha,\beta\in\mathrm{Hom}_H(A,A)$ 构造 BiHom-结合 H-伪代数$(A,\mu(\alpha\otimes\beta),\alpha^{\sim}\alpha,\beta^{\sim}\beta)$. 最后，给出了在 2 个 BiHom-结合 H-伪代数的张量积空间 $A\otimes B$ 上构造 BiHom-结合 H-伪代数的方法.

关键词：BiHom-结合 H-伪代数；Yau 扭曲；张量积 BiHom-结合 H-伪代数

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