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Exponential synchronization for delayed nonlinear Schrödinger equation and applications in optical secure communication

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Abstract: For further exploring the confidentiality of optical communication, exponential synchronization for the delayed nonlinear Schrödinger equation is studied. It is possible for time-delay systems to generate multiple positive Lyapunov exponents without the limitation of system dimension. Firstly, the homoclinic orbit analysis is carried out by using the bifurcation theory, and it is found that there are two homoclinic orbits in the system. According to the corresponding relationship, solitary waves also exist in the system. Secondly, the Melnikov method is used to prove that homoclinic orbits can evolve into chaos under arbitrary perturbations, and then chaotic signals are used as the carriers of information transmission. The Lyapunov exponent spectrum, phase diagram and time series of the system also prove the existence of chaos. Thirdly, an exponential synchronization controller is designed to achieve the chaotic synchronization between the driving system and the response system, and it is proved by the Lyapunov stability theory. Finally, the error system is simulated by using MATLAB, and it is found that the error tends to zero in a very short time. Numerical simulation results demonstrate that the proposed exponential synchronization scheme can effectively guarantee the chaotic synchronization within 1 s.

Key words: secure communication; Melnikov method; nonlinear Schrödinger equation; exponential synchronization
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In recent years, optical secure communication based on chaotic synchronization has become an active area of research in the cross-convergence of international nonlinear science and information science[1-3]. Scientists proposed a variety of chaotic synchronization methods and applied them to secure communications. Kocamaz et al.[4] studied the passive control-based chaos synchronization with circuit design for secure communication and found that a single state passivity-based synchronization signal can be effectively used for secure data communication applications in the real environment. Mata-Machuca and Aguilar-Lopez[5] studied the adaptative synchronization of complex dynamical networks with fractional-order nodes and its application in secure communications employing chaotic parameter modulation. Durdu and Uyaroǧlu[6] studied the exponential stability for encrypts of signals in the synchronization of chaotic systems. Smaoui et al.[7] proposed a novel secure communication scheme based on the Karhunen Loéve decomposition and the synchronization of a master and a slave hyperchaotic Lü system. However, the low-dimensional chaotic system is easily deciphered, and hyperchaotic systems are not suitable for practical applications due to their complex structure[8].

With the discovery of delayed chaotic systems, new ideas are proposed for secure communication. Since the time-delayed chaotic system can generate multiple positive Lyapunov exponents and is not limited by the system dimension, a large number of studies on time-delayed chaotic systems have been carried out. Oden et al.[9] proposed a chaos communication scheme based on a chaotic optical phase carrier generated with an optoelectronic oscillator with nonlinear time-delay feedback. Maheri and Arifin[10] put forward the synchronization of chaotic systems which is defined based on the exponential stability for the encrypts of signals. Abd et al.[11] developed a new cascade-coupled chaotic synchronization model based on the first-order nonlinear time-delayed chaotic system. However, as one of the best communication models, the delayed nonlinear Schrödinger equation is rarely used for secure communication. In fact, the propagation model of optical fibers is described by the nonlinear Schrödinger equation, so the optical fiber transmission system cannot be separated from the nonlinear Schrödinger equation. Yin et al.[12] proposed an optical secure communication scheme based on chaos synchronization, which provides a theoretical basis for studying the delayed nonlinear Schrodinger equation. Therefore, this paper will study exponential synchronization for the delayed nonlinear Schrödinger equation.

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1 Chaos Generation

This paper devises optical secure communication based on the perturbed nonlinear Schrödinger equation[13-15] by chaos synchronization. The nonlinear Schrodinger equation is

$$\mathrm{i}u_t + u_{xx} + \alpha\ |u|^2 u + \mathrm{i}[\gamma_1 u_{xxx} + \gamma_2\ |u|^2 u_x + \gamma_3\ (|u|^2)_x u] = \mu u \cos\omega(x - ct) \tag{1}$$

where $\gamma_i\ (i = 1,2,3)$ are real parameters; α is a positive constant; $\mu, \omega \geqslant 0$ denote the amplitude and the frequency of the parametric excitation, respectively. More details for the model can be seen in Refs. [13,15].

Suppose that Eq. (1) has traveling wave solutions in the form

$$\mu(x,t) = a(\xi)\exp(-\mathrm{i}\Omega t) \qquad \xi = x - ct \qquad c > 0 \tag{2}$$

where c represents the transmission speed of wave.

By the method of Ref. [13], we have

$$\gamma_1 \varphi'' - c\varphi + \lambda\varphi^3 = \mu\varphi\cos(\omega\xi) \tag{3}$$

where $\lambda = \frac{1}{3}\gamma_2 + \frac{2}{3}\gamma_3$ refers to the parameter of linear and nonlinear terms. Eq. (3) is a fiber-optic signal transmission system in an ideal environment.

By setting $\gamma_1 = 1$ and using transformation $\varphi = x_1, x_1' = x_2$, Eq. (3) can be rewritten as a set of two autonomous differential equations as below:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= (u\cos(\omega\xi) + c)x_1 - \lambda x_1^3 \end{aligned}\right\} \tag{4}$$

Next, we consider the delayed system (4) which can be written as

$$\left.\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2 &= (u\cos(\omega\xi) + c)x_1(t-\tau) - \lambda x_1(t)^3 \end{aligned}\right\} \tag{5}$$

If $u = 0$, Eq. (5) is changed into

$$\left.\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2 &= cx_1(t-\tau) - \lambda x_1(t)^3 \end{aligned}\right\} \tag{6}$$

1.1 Analysis of homoclinic orbits

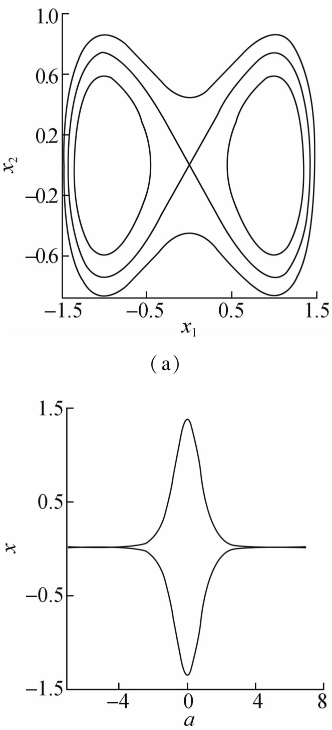
It is known from the calculation that Eq. (6) has three equilibrium points, $E_1\left(-\sqrt{\frac{c}{\lambda}}, 0\right)$, $E_2\left(\sqrt{\frac{c}{\lambda}}, 0\right)$ and $E_3(0,0)$. Let $\boldsymbol{J}_{E_i}$ be the Jacobian matrix for these equilibrium points, then we obtain

$$\boldsymbol{J}_{E_i} = \begin{bmatrix} 0 & 1 \\ c - 3\lambda x_1^3 & 0 \end{bmatrix}$$

It is easy to find that its eigenvalues are $\lambda_{1,2}(\boldsymbol{J}_{E_i}) = \pm\sqrt{-2c}$ and $\lambda_3(\boldsymbol{J}_{E_3}) = \pm\sqrt{c}$. Therefore, we conclude that E_1 and E_2 are the center equilibriums and E_3 is a saddle point for any $c > 0$. Furthermore, Eq. (6) has the following Hamiltonian function:

$$H(x_1, x_2) = \frac{1}{2}x_2^2 - \frac{c}{2}x_1^2 + \frac{1}{4}\lambda x_1^4 = h \tag{7}$$

where h is a constant. It is noted that the Hamiltonian function is composed of two homoclinic orbits at point E_3. According to the bifurcation theory[13], Eq. (6) has two optical solitons followed by two homoclinic orbits: the positive one achieves its crest at $x = \sqrt{2c/\lambda}$, and the negative one has a valley at $x = -\sqrt{2c/\lambda}$ (see Fig. 1 as $\lambda = 2.2, c = 2.2$).



(b)

Fig. 1 The profile of soliton in the unperturbed system. (a) Phase portrait of Eq. (6); (b) The profile of solitary

1.2 Analysis of chaos

The optical soliton always turns into chaos under the perturbation formed as Eq. (1). We cannot ignore such a fact that the solitary wave solution of the nonlinear evolution equation is equivalent to the homoclinic orbit of the corresponding dynamic system. In this section, we will prove that homoclinic orbit evolves into chaos under external disturbance by using the Melnikov method.

Unperturbed homoclinic orbits can be written as $(a,b) = (a_0(t), b_0(t))$. According to the Melnikov method, the Melnikov function for Eq. (3) is defined as

$$M(t_0) = \int_{-\infty}^{+\infty} b_0(t)\mu\cos(\omega(t + t_0))\,\mathrm{d}t = \frac{\mu\omega}{2}\sin\omega t_0 I \tag{8}$$

where $I = \int_{-\infty}^{+\infty} a_0^2\cos(\omega t)\,\mathrm{d}t$ is a function of ω.

According to the Melnikov method, chaos occur if $M(t_0) = 0$ and $M'(t_0) \neq 0$ for some t_0. We observe that $M(0) = 0$ and $M'(0) \neq 0$. Hence, the optical soliton can turn into chaos under the perturbation.

To verify the above fact, we will investigate the Lyapunov exponents, phase portraits and time series of Eq. (6). The parameters of Eq. (6) used for simulations are listed as follows: $\omega = 0.27, \tau = 0.1, c = 0.8, \lambda = 2.2$, $\mu = 0.47$. Then the Lyapunov exponents are shown in Fig. 2, the phase portrait is shown in Fig. 3, and the time series are shown in Fig. 4. As can be seen in Fig. 2, the Lyapunov exponents are positive so that the motion of Eq. (3) is chaotic. The phase portrait and corresponding time series also show the existence of chaos.

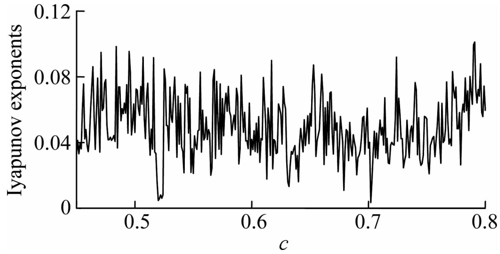


Fig. 2 Lyapunov exponents versus c of system (5)

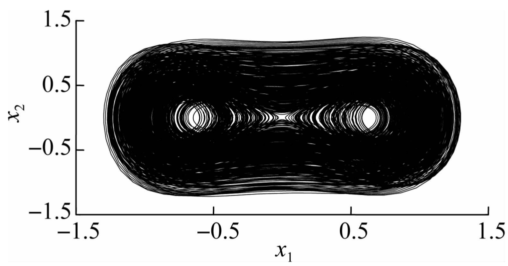


Fig. 3 Phase portrait x_1-x_2

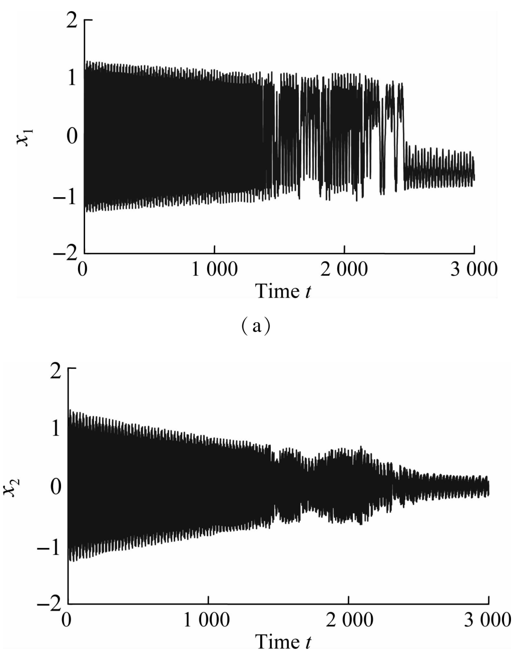


Fig. 4 The time series of x_1 and x_2. (a) x_1; (b) x_2

2 Problem Description

2.1 Exponential stability

In this section, we consider the delayed system (4). For ease of description, the delayed system can be written as

$$\left.\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= a\cos(\omega\xi)x_1(t-\tau) + bx_1(t) - cx_1(t)^3 \end{aligned}\right\} \tag{9}$$

Let Eq. (9) for the drive system and the corresponding response system be as below:

$$\left.\begin{aligned} \dot{y}_1(t) &= y_2(t) + \mu_1 \\ \dot{y}_2(t) &= \hat{a}\cos(\omega\xi)y_1(t-\tau) + \hat{b}y_1(t) - \hat{c}y_1(t)^3 + \mu_2 \end{aligned}\right\} \tag{10}$$

where $\hat{a}$, $\hat{b}$ and $\hat{c}$ are parameters to be estimated; μ_1 and μ_2 are the controllers to be designed to achieve exponential synchronization between the drive system and the response system.

The error signal is selected as $e(t) = y(t) - x(t)$, and the controller is the error feedback control as below:

$$\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{e}(t) = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}\begin{bmatrix} e_1(t) & 0 \\ 0 & e_2(t) \end{bmatrix} \tag{11}$$

where $\boldsymbol{K}$ is the control gain matrix which makes the trajectory between the driving chaotic system (9) and the response chaotic system (10) achieve exponential synchronization.

Definition 1 The system is exponentially stable if $\theta > 0, \rho > 0$ and for each $\boldsymbol{e}(t)$ solution of the system, the following inequality holds:

$$\|\boldsymbol{e}(t)\| \leqq \rho \mathrm{e}^{-\theta t} \sup_{-\tau \leqslant \lambda \leqslant 0} \|\boldsymbol{e}(\lambda)\| \qquad \forall t > 0 \tag{12}$$

where θ is the exponential convergence rate; $\|\cdot\|$ denotes the Euclidean norm of a vector.

The controller is selected as the form of (11) and the error variables between systems (9) and (10) are

$$\left.\begin{aligned} e_1(t) &= y_1(t) - x_1(t) \\ e_2(t) &= y_2(t) - x_2(t) \\ e_1(t-\tau) &= y_1(t-\tau) - x_1(t-\tau) \end{aligned}\right\} \tag{13}$$

Then, we obtain the error system:

$$\left.\begin{aligned} \dot{e}_1(t) &= e_2(t) + k_1e_1(t) \\ \dot{e}_2(t) &= \cos(\omega t)[(\hat{a}-a)y_1(t-\tau) + ae_1(t-\tau)] + \\ &\quad [(\hat{b}-b)y_1 + be_1(t)] - [(\hat{c}-c)y_1^3(t) + \\ &\quad ce_1(t)(y_1^2(t) + y_1(t)x_1(t) + x_1^2(t))] \end{aligned}\right\} \tag{14}$$

Furthermore, let $\boldsymbol{e}(\lambda) = \{e_1(\lambda), e_2(\lambda)\}^{\mathrm{T}} = \boldsymbol{\varphi}(\lambda)$, $\lambda \in [-\tau, 0]$.

2.2 Synchronization based on exponential stability

Theorem 1 The response system(10) will synchronize exponentially with the driving system (9) for any initial values. If $\boldsymbol{\varphi}(\lambda) \neq 0$, the exponential synchronization controls are chosen as control laws and the following matrix inequalities hold:

$$\begin{bmatrix} 2p\theta + q + 2pk_1 & p + pb \\ p + pb & \theta + q^{-1}\mathrm{e}^{2\theta\tau}p^2a^2 + 2pk_2 \end{bmatrix} < 0$$

The undetermined parameters of response system (10) are as below:

$$\left.\begin{aligned} \dot{\hat{a}} &= -\mathrm{e}^{2\theta t}pa\cos(\omega t)y_1(t-\tau)e_2(t) \\ \dot{\hat{b}} &= -\mathrm{e}^{2\theta t}py_1(t)e_2(t) \\ \dot{\hat{c}} &= \mathrm{e}^{2\theta t}py_1^3(t)e_2(t) \end{aligned}\right\} \tag{15}$$

where $p, q, \theta > 0$.

Proof Let

$$V(e(t)) = \mathrm{e}^{2\theta t}p(e_1^2(t) + e_1^2(t)) + \int_{t-\tau}^{t} \mathrm{e}^{2\theta s}qe_1^2(s)\,\mathrm{d}s + (e_a^2(t) + e_b^2(t) + e_c^2(t)) \tag{16}$$

where $e_a = \hat{a} - a, e_b = \hat{b} - a$ and $e_c = \hat{c} - c$ are the estimation errors of $\hat{a}, \hat{b}$ and $\hat{c}$, respectively. The time derivatives of function (16) for the error system (14) and control laws (11) are

$$\begin{aligned} \dot{V}(e(t)) = {} & \mathrm{e}^{2\theta t}[2\theta p(e_1^2 + e_2^2) + 2p(\dot{e}_1e_1 + \dot{e}_2e_2) + qe_1^2 - \\ & \mathrm{e}^{-2\theta\tau}qe_{1\tau}^2] + 2(\dot{e}_ae_a + \dot{e}_be_b + \dot{e}_ce_c) = \\ & \mathrm{e}^{2\theta t}[2\theta p(e_1^2 + e_2^2 + qe_1^2) + 2p(e_1e_2 + k_1e_1^2) + \\ & 2p\cos(\omega t)((\hat{a} - a)y_{1\tau}e_2 + ae_{1\tau}e_2) + \\ & ((\hat{b} - b)y_1e_2 + be_1e_2) - ((\hat{c} - c)y_1^3e_2 + \\ & ce_1e_2(y_1^2 + y_1x_1 + x_1^2) + k_2e_2^2) - \mathrm{e}^{-2\theta\tau}qe_{1\tau}^2] + \\ & 2e_a\dot{\hat{a}} + 2e_b\dot{\hat{b}} + 2e_c\dot{\hat{c}} \end{aligned} \tag{17}$$

Substituting Eq. (14) into Eq. (17) yields

$$\begin{aligned} \dot{V}(e(t)) = {} & \mathrm{e}^{2\theta t}[(2\theta p + q + 2pk_1)e_1^2 + (2\theta p + 2pk_2)e_2^2 + \\ & (2p + 2pb - 2pc(y_1^2 + y_1x_1 + x_1^2))e_1e_2 + \\ & 2pae_{1\tau}e_2\cos(\omega t) - \mathrm{e}^{-2\theta\tau}qe_{1\tau}^2] \leqslant \\ & \mathrm{e}^{2\theta t}[(2\theta p + q + 2pk_1)e_1^2 + (2\theta p + 2pk_2)e_2^2 + \\ & (2p + 2pb)e_1e_2 + q^{-1}\mathrm{e}^{2\theta\tau}p^2a^2e_2^2] \leqslant \\ & [(2\theta p + q + 2pk_1)e_1^2 + (2\theta p + 2pk_2 + \\ & q^{-1}\mathrm{e}^{2\theta\tau}p^2a^2)e_2^2 + (2p + 2pb)\|e_1e_2\|] = \\ & \boldsymbol{e}^{\mathrm{T}}\boldsymbol{A}\boldsymbol{e} \end{aligned} \tag{18}$$

where

$$\boldsymbol{A} = \begin{bmatrix} 2p\theta + q + 2pk_1 & p + pb \\ p + pb & 2p\theta + q^{-1}\mathrm{e}^{2\theta\tau}p^2a^2 + 2pk_2 \end{bmatrix}$$

From Theorem 1, $\boldsymbol{A} < 0$, and we obtain $\dot{V}(\boldsymbol{e}(t)) < 0$. Therefore, $V(\boldsymbol{e}(t)) < V(\boldsymbol{e}(0))$.

$$V(\boldsymbol{e}(0)) = p(e_1^2 + e - 2^2) + \int_{-\tau}^{0} \mathrm{e}^{2\theta s}qe_1^2(s) + (e_a^2 + e_b^2 + e_c^2) \leqslant p\|\boldsymbol{\varphi}\|^2 + q\frac{1}{2\theta}\|\boldsymbol{\varphi}\|^2 + (e_a^2 + e_b^2 + e_c^2) \tag{19}$$

For $\eta > 0$,

$$V(\boldsymbol{e}(0)) \leqslant \eta \sup_{-\tau \leqslant \lambda \leqslant 0} \|\boldsymbol{e}(\lambda)\|^2 \tag{20}$$

For

$$V(\boldsymbol{e}(t)) \geqslant \mathrm{e}^{2\theta}p\|\boldsymbol{e}(\lambda)\|^2 \tag{21}$$

Therefore,

$$\mathrm{e}^{2\theta}p \leqslant \eta \sup_{-\tau \leqslant \lambda \leqslant 0} \|\boldsymbol{e}(\lambda)\|^2 \tag{22}$$

That is

$$\|\boldsymbol{e}(t)\| \leqslant \rho\mathrm{e}^{-\theta t} \sup_{-\tau \leqslant \lambda \leqslant 0} \|\boldsymbol{e}(\lambda)\| \tag{23}$$

where $\rho = \eta p^{-1}$. Therefore, in this case, the synchronization of driving system (9) and response system (10) are sufficiently achieved.

3 Results and Discussion

3.1 Simulation

In this section, we will analyze the performance of the delay chaotic secure communication system. System (12) was simulated in Matlab-Simulink and the results are shown in Fig. 5. The initial conditions of the response system and driving system are $\boldsymbol{x}_0 = [0.03 \quad 0.01]^{\mathrm{T}}$, $\boldsymbol{y}_0 =$

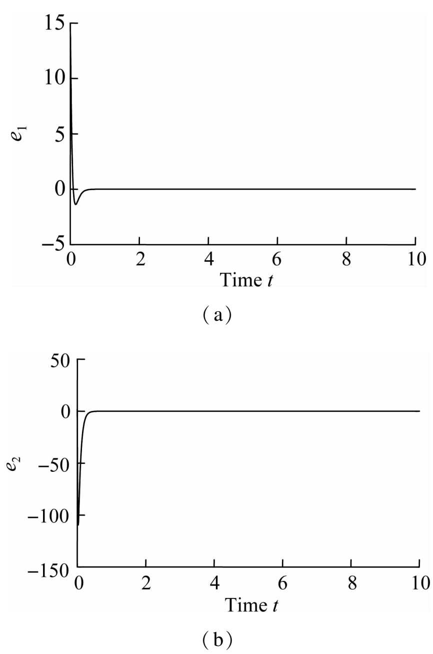


Fig. 5 The error curves. (a) Error e_1; (b) Error e_2

$[15 \quad 2]^{T}$, respectively. The initial value of the parameters is chosen as $\hat{a}=0.15$, $\hat{b}=3.1$, $\hat{c}=3.1$. When the exponential convergence rate $\theta=0.2$, we obtain $p=2.6$, $q=0.52$, $k_1=-15.5$, $k_2=-13.8$.

Fig. 5 shows the synchronization difference (error) between systems (9) and (10). It is obvious that the error system tends to zero very fast (within 1s), indicating that the signal is sent and received faster with a better accuracy.

3.2 Discussion

Undoubtedly, there are abundant research results regarding chaotic secure communication, especially for the increasing number of alternative communication models. Considering that the propagation model of optical fibers is described by the nonlinear Schrchrödinger equation as we mentioned in section 1, the nonlinear Schrchrödinger equation with time delay is studied and applied to secure communication in this paper. Firstly, the optical fiber communication model studied in this paper is different from that in Refs. [9,11-12]. This paper studied the delay nonlinear Schrchrödinger equation while Ref. [9] studied the chaos communication scheme based on a chaotic optical phase carrier generated with an optoelectronic oscillator. Ref. [11] studied the chaotic synchronization model based on the first-order nonlinear time-delayed Lü system. Secondly, the chaotic phenomena in this paper (see Fig. 3) are different from those in the famous Lorenz system, Chen system and Lü system. Thirdly, the exponential synchronization controller designed in this paper is similar to that in Ref. [16]. Finally, the numerical simulation results show that systems (9) and (10) achieve synchronization in a very short time. By comparing with Refs. [10,12], the synchronization speed in this paper is the fastest.

4 Conclusions

1) In this paper, the optical secure communication based on the delayed nonlinear Schrödinger equation was studied. Chaotic signals are obtained with periodic perturbation. Lyapunov exponential spectra, phase portrait and time series are used to prove the existence of chaotic signals.

2) The sufficient criteria for the exponential synchronization of time-delay chaotic systems were obtained by employing the Lyapunov stability theory and linear matrix inequality technology.

3) Despite the differences between the driving system and the response system, the exponential synchronization can still be achieved by the driving-responding synchronization method. Numerical simulation shows that the error tends to zero in a very short time (within 1 s).

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时滞非线性薛定谔方程的指数同步及其在光纤保密通信中的应用

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摘要:为了进一步探究光纤通信的保密特性,研究了时滞非线性薛定谔方程的指数同步问题. 时滞混沌系统具有产生多个正的李雅普诺夫指数的可能性,且不受系统维数的限制. 首先利用分岔理论进行了同宿轨分析,研究发现系统存在 2 条同宿轨,根据对应关系,系统存在孤立波. 其次,利用 Melnikov 方法证明了同宿轨在任意扰动下可以演变为混沌,进而将混沌信号作为信息传输的载体. 系统的 Lyapunov 指数谱图、相图以及时间序列分析同样证明了混沌的存在性. 再次,设计了指数同步控制器, 实现驱动系统和响应系统的混沌同步,并利用 Lyapunov 稳定性理论进行了证明. 最后,利用 MATLAB 对误差系统进行了数值仿真,发现误差很快趋向于零. 研究结果表明,所提出的指数同步方案可在 1 s 内实现驱动系统和响应系统的同步.

关键词:保密通讯;Melnikov 方法;非线性薛定谔方程;指数同步

中图分类号:O231.2