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A uniaxial tensile constitutive equation of high-ductility cementitious composites

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Abstract: In order to establish the constitutive relationship of high-ductility cementitious composites (HDCCs) under uniaxial tensile load and to guide the structural design of HDCCs, based on the analysis of the existing uniaxial tensile constitutive relationship and ideal elastoplastic linear strain-hardening model, a bilinear tensile constitutive equation of HDCCs was proposed. The points of nominal initial cracking and nominal maximum stress were adopted as control points of the line segment, and the constitutive relationship of HDCCs was established. Five series of uniaxial tensile stress-strain curves of HDCCs were combined to perform an experimental application of the constitutive equation, along with an analysis of the key parameters. The experimental results confirm the ability of the constitutive equation to overcome the problem of insufficient or excessive redundancy of existing models in terms of calculation bearing capacity. Specifically, the measured maximum stress value is larger than the nominal value, and the ratio between the two values ranges from 1.08 to 1.22. Additionally, the tensile strain at the softening point obtained by fitting a straight line with the valley points of the strain-hardening stage curve is greater than or equal to the tensile strain at the measured maximum stress point and the ratio of the fitted values to the measured values ranges from 1.00 to 1.19.

Key words: uniaxial tensile; constitutive equation; simplified model; high-ductility cementitious composites

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High-ductility cementitious composites (HDCCs) are a kind of high-performance fiber-reinforced cement-based composites exhibiting high ductility under tensile load[1]. Their application prospects are broad, which mainly include concrete structure repair and seismic reinforcement, as well as other projects[2]. As a new type of building material, HDCCs inevitably employ a constitutive relationship to design, calculate, and check its structural bearing and deformation capacities in the application process. In this regard, establishing a scientific and reasonable constitutive relationship is a precursor for the large-scale application of HDCCs. Li et al. [3] investigated the main factors affecting the compressive properties of HDCCs and their uniaxial compressive constitutive equation. Deng et al. [4] established the damage constitutive model of highly ductile concrete based on the double-parameter damage threshold. Maalej et al. [5-6] abstracted and simplified the uniaxial tensile stress-strain curve of HDCCs. Kanda et al. [7] proposed a bilinear uniaxial tensile constitutive model. Such studies included systematic investigations accounting for the constitutive relationship of HDCCs under uniaxial compression, whereas those accounting for the constitutive relationship under uniaxial tension remain relatively rare. The unique multi cracking and strain-hardening characteristics of HDCCs make their constitutive relationship significantly different from that of ordinary concrete or fiber-reinforced concrete, which logically implies that the commonly used tensile constitutive model for concrete is not suitable for HDCCs. In order to make full use of the highly ductile characteristics of HDCCs for their development and feasible applications, their constitutive relationship under uniaxial tension load should be urgently established for structural design and calculation guidance[8-9]. Based on the ideal elastoplastic linear strengthening model and in accordance with the summary of the already existing uniaxial tensile constitutive relationship, this paper highlights the proposal of a constitutive equation for HDCCs that can be used as a guide for the structural bearing capacity design and deformation calculation of HDCCs.

1 Characteristics of the HDCC Uniaxial Tensile Stress-Strain Curve

HDCCs have multiple discernible cracks and strain-hardening characteristics under uniaxial tension. More specifically, their uniaxial tension stress-strain curve dis-

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plays distinct characteristics from those of ordinary or fiber-reinforced concrete and is typically divided into three stages, as illustrated in Fig. 1.

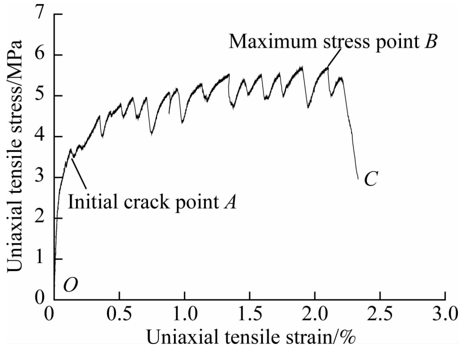

Fig. 1 Representative uniaxial tensile stress-strain curve of HDCCs

Initial stage: Elastoplastic phase (*OA* segment). The specimen exhibits elastic-plastic stress and strain from the loading inset to the occurrence of the first crack (i. e., the *OA* section). Strain increases gradually with the increase in the load from the origin point *O*, and the two parameters are observed to exhibit a certain linear relationship with each other. At the end of the *OA* segment, any further increase in the load results in the gradual transition of deformation from elastic to plastic until preliminary cracking. Deformation of the *OA* segment is mainly elastic and is supplemented by plastic deformation. In Fig. 1, *A*, σ_c, and ε_c correspond to the initial crack point, initial crack stress, and initial crack strain, respectively.

Secondary stage: Steady-state cracking or strain-hardening phase (*AB* section). At this stage, both load and deformation are continuously increased, and stress is transferred to the matrix from the bridging fibers at the cracks, thereby creating new cracks. Nevertheless, crack propagation is restrained by the fiber bridging stress, thereby maintaining steady-state cracking. With the further increases of load, the number and width of the cracks increase, until the bearing capacity of the HDCC ultimately reaches its maximum. At this phase, point *B* corresponds to the maximum tensile stress σ_u and tensile strain ε_u.

Final stage: Strain-softening stage (*BC* section). The main crack expands and runs through the entire cross section of the specimen. The bearing capacity then decreases sharply, leading to failure.

2 Simplification of Uniaxial Tensile Stress-Strain Curve of HDCCs

Maalej et al. [5,7] used the above analysis to simplify the tensile relationship curve of HDCCs. In this process, the *OA* and *AB* segments are simplified into straight lines, whereas the *BC* segment can be ignored in practical applications because of the loss of the bearing capacity, as shown in Fig. 2(a). The uniaxial tension stress-strain curve of HDCCs is further simplified into an *OA-AB* bilinear model, as shown in Fig. 2(b). According to the characteristics of this simplified stress-strain bilinear model, the ideal elastic-plastic linear hardening model in elastoplastic mechanics can be used to establish the tension constitutive equation.

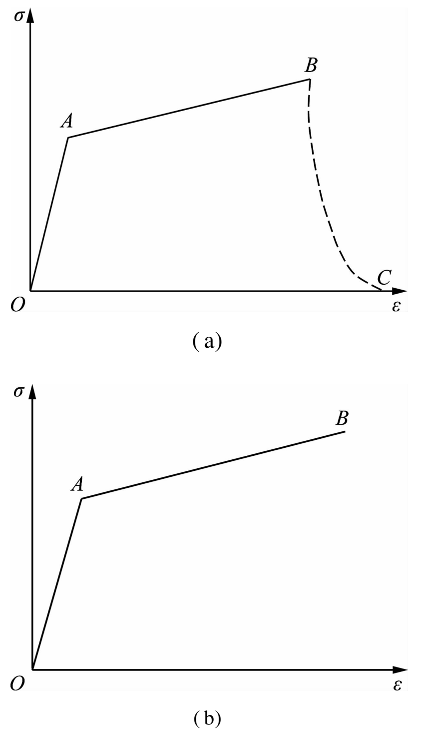

Fig. 2 Simplified uniaxial tensile stress-strain curves for HDCCs[5,7]. (a) Three-segment line; (b) Bilinear

Accordingly, the ideal elastic-plastic linear hardening model of HDCCs can be obtained in two ways.

First, the initial crack point $A(\varepsilon_c, \sigma_c)$ and the maximum tensile stress point $B(\varepsilon_u, \sigma_u)$ are determined. By connecting the origin, the initial crack point, and the maximum tensile stress point, a two-stage linear equation, as shown in Fig. 3(a) $\left(\text{i. e., } \sigma(\varepsilon) = \frac{\sigma_c}{\varepsilon_c}\varepsilon \text{ and } \sigma(\varepsilon) = \sigma_c + \frac{\sigma_u - \sigma_c}{\varepsilon_u - \varepsilon_c}(\varepsilon - \varepsilon_c)\right)$ is obtained, where ε_c, ε_u, σ_c, and σ_u are all measured values.

Secondly, by fitting the equation of straight line in the elastic stage $\left(\text{i. e., } \sigma(\varepsilon) = \frac{\sigma_{nc}}{\varepsilon_c}\varepsilon\right)$, the nominal initial crack stress σ_{nc} and the nominal initial crack point $A(\varepsilon_c, \sigma_{nc})$ are determined. Connecting $A(\varepsilon_c, \sigma_{nc})$ with the actual maximum tensile stress point $B(\varepsilon_u, \sigma_u)$ yields the straight line and its equation, $\sigma(\varepsilon) = \sigma_{nc} + \frac{\sigma_u - \sigma_{nc}}{\varepsilon_u - \varepsilon_c}(\varepsilon - \varepsilon_c)$, in the strain-hardening stage, as shown in Fig. 3 (b), where ε_c, ε_u, and σ_u are measured values, and the nominal initial crack stress σ_{nc} is a fitted value.

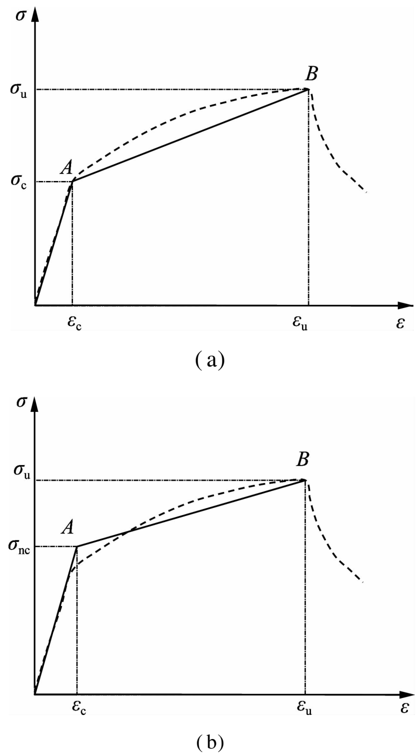

Fig. 3 Simplified uniaxial tensile stress-strain curves of HDCCs[5]. (a) Initial crack stress is a measured value; (b) Initial crack stress is a fitted value

The common problems associated with these two methods include the followings:

1) The stress and strain at the initial crack point are difficult to determine. The coordinates of point *A* depend on the appearance of the initial crack. However, the initial crack normally has a very small width (approximately 100 microns), and the stress drop is not always observable with the initial crack appearance. Thus, there are some subjective factors involved in determining the stress and strain of initial cracks due to the limitation of test accuracy.

2) Taking the measured maximum stress point as the control point of the equation restricts the redundancy of structural design calculation.

3) As in 1), the coordinates of point *B* are not easily determined under the assumption that the maximum stress point and the softening point do not coincide.

To obtain sufficient design redundancy, the ideal elastic-plastic linear hardening model can be further simplified to an ideal elastic-plastic model, as shown in Fig. 4. Here, the initial crack point $A(\varepsilon_c, \sigma_c)$ or the nominal initial crack point $A(\varepsilon_c, \sigma_{nc})$ is first determined. During the elastic stage, $\sigma(\varepsilon) = \dfrac{\sigma_c}{\varepsilon_c}\varepsilon$ or $\sigma(\varepsilon) = \dfrac{\sigma_{nc}}{\varepsilon_c}\varepsilon$. After the initial crack point *A*, strain increases without any change in stress (i. e., $\sigma(\varepsilon) = \sigma_c$ or $\sigma(\varepsilon) = \sigma_{nc}$). That is, the model only considers the increase of the HDCC's strain in the strain-hardening stage, but not the increase of the stress caused by the fiber bridging effect, hence making it a conservative model for structural design. The Japan Society of Civil Engineers' code, "Recommendations for Design and Construction of High Performance Fiber-Reinforced Cement Composites with Multiple Fine Cracks," adopts this design concept[10].

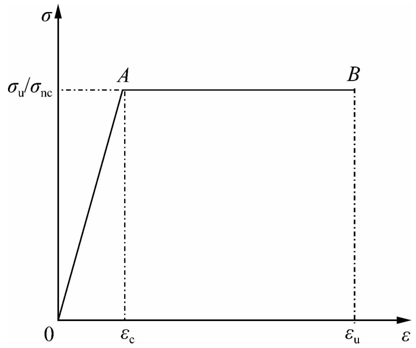

Fig. 4 Ideal elastic-plastic model for HDCC's uniaxial tension

3 Establishment of a Simplified Model of Uniaxial Tensile Constitutive Relationship for HDCCs

Accordingly, a new constitutive equation for HDCC tension is proposed on the basis of the ideal elastic-plastic linear hardening model as shown in Fig. 5, which is fixated on solving the problems of difficulty in determining the initial crack point and the insufficient or excessive bearing capacity in the already existing bilinear models. The equation constitutes the following characteristics:

1) A valley point of stress is selected during the strain-hardening phase, for straight line fitting.

The valley point of the strain-hardening curve is selected for fitting, not only making full use of the strain-hardening characteristics of HDCCs, but also retaining the appropriate redundancy.

2) The initial crack point is determined from the intersection of two straight lines, and the nominal maximum stress value σ_{nu} is determined by the tensile strain value ε_u corresponding to the measured maximum stress value σ_u.

The point of intersection of the fitting lines of the elastic segment and the steady cracking stage is the newly defined nominal initial crack point $(\varepsilon_{nc}, \sigma_{nc})$. In the steady-state

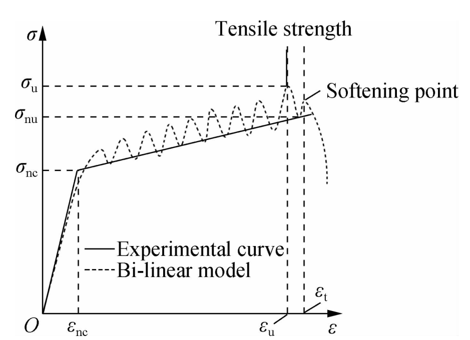

Fig. 5 A schematic diagram of the HDCC's uniaxial tensile constitutive relationship

cracking stage, the nominal maximum stress point is (ε_u, σ_{nu}). The uniaxial tension bilinear constitutive equation of HDCCs is more appropriately expressed as follows:

$$\sigma(\varepsilon)=\begin{cases}\dfrac{\sigma_{nc}}{\varepsilon_{nc}}\varepsilon & 0\leqslant\varepsilon\leqslant\varepsilon_{nc}\\[2ex] \sigma_{nc}+\dfrac{\sigma_{nu}-\sigma_{nc}}{\varepsilon_u-\varepsilon_{nc}}(\varepsilon-\varepsilon_{nc}) & \varepsilon_{nc}<\varepsilon\leqslant\varepsilon_u\end{cases} \tag{1}$$

4 Application Example of HDCC Uniaxial Tensile Bilinear Constitutive Equation

Five groups of HDCC uniaxial tension stress-strain curves[11], numbered H1 to H5, are selected to demonstrate the application of the bilinear model of Eq. (1). The straight lines of the elastic section and the strain-hardening stage are obtained by fitting, as shown in Fig. 6. Note that such good consistency displayed by the fitting straight lines with the development trend of the measured curve can be used to characterize the elastic deformation and strain hardening of HDCCs. Moreover, the nominal initial crack strain ε_{nc}, nominal initial crack stress σ_{nc}, and nominal maximum tensile stress σ_{nu} of HDCCs are obtained from the intersection of two straight lines and the maximum stress point on the stress-strain curve, as depicted in Tab. 1.

During the HDCC tensile test, the bearing capacity was observed to decrease slightly following the appearance of the first crack, mainly due to cracking of the matrix[12]. Additionally, a peak shape was apparently visible on the stress-strain curve. On this basis, the primary peak point was defined as the initial crack point of the HDCC. For the tension test, the softening point was defined as the point where the bearing capacity decreased significantly with the expansion of the main crack's width, whereas the strain corresponding to the softening point was defined as the ultimate tensile strain. The initial crack stress, initial crack strain, and ultimate tensile strain of the five groups of HDCCs are summarized in Tab. 1 accordingly.

A comparison of the measured values with the nominal values of the bilinear model (see Tab. 2) apparently indicated a significant difference in both results for the ratio of the initial cracking stress, whereas the initial cracking strain was higher in the measured values than that in the calculated ones. Moreover, an analysis of the typical uniaxial tension stress-strain curves of HDCCs (see Fig. 1) confirmed that the deformation on the front part of segment *OA*, shown as a straight line, is elastic, whereas its tail end enters a plastic deformation stage, as the straight line assumes a convex curve. Furthermore, as the nominal initial crack point was the bilinear intersections, such point was bound to appear on the left side of the actual initial crack point (see Fig. 5), which necessitates the nominal initial crack strain being bound to be smaller than the measured value. Similarly, the nominal initial crack stress

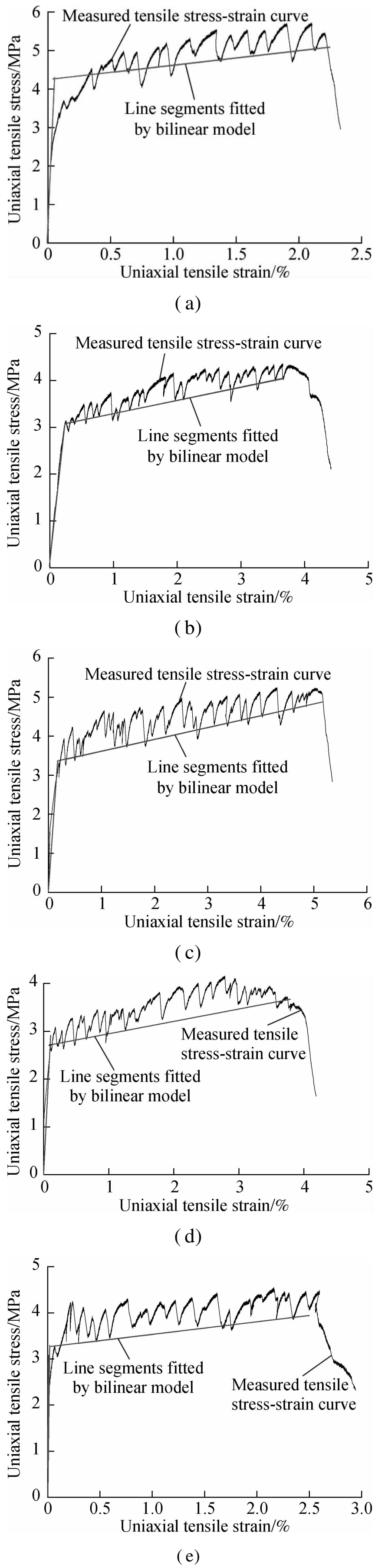

Fig. 6 Measured uniaxial tensile stress-strain curve and bilinear model for HDCCs. (a) H1; (b) H2; (c) H3; (d) H4; (e) H5

Tab. 1 Measured and fitting values of control points for HDCC uniaxial tensile bilinear constitutive equations

Serial number	Initial crack stress σ_c/MPa	Nominal initial crack stress σ_{nc}/MPa	Initial crack strain ε_c/%	Nominal initial crack strain ε_{nc}/%	Maximum tensile stress σ_u/MPa	Nominal maximum tensile stress σ_{nu}/MPa	Tensile strain at maximum stress point ε_u/%	Ultimate tensile strain ε_t/%
H1	3.37	4.26	0.09	0.05	5.76	5.03	2.10	2.19
H2	3.12	3.08	0.25	0.24	4.35	4.03	3.65	3.78
H3	3.32	3.37	0.20	0.17	5.23	4.84	5.05	5.04
H4	2.71	2.71	0.11	0.10	4.15	3.39	3.41	3.85
H5	3.28	3.27	0.06	0.02	4.53	3.85	2.16	2.58

Tab. 2 Ratios of measured to nominal values of HDCCs

Serial number	Ratio of measured value to nominal value			Tensile strain ratio between softening point and maximum stress point
	Initial crack stress	Initial crack strain	Maximum stress	
H1	0.79	1.68	1.15	1.04
H2	1.02	1.06	1.08	1.04
H3	0.98	1.14	1.08	1.00
H4	1.00	1.07	1.22	1.13
H5	1.00	3.00	1.18	1.19

was bound to be either larger than the actual value or smaller than the measured value, as it was mainly dependent on the slope of the two straight lines. Namely, a larger linear slope of the elastic section will assume a smaller initial crack stress; otherwise, a larger linear slope of the strain-hardening section will assume a larger initial crack stress.

The ratio of the measured maximum stress to the nominal value of the five groups of HDCCs ranges from 1.08 to 1.22, which is greater than 1, mainly because the straight line in the stress-hardening stage was fitted from the stress valley of the stress-strain curve. As such, the measured value was bound to be larger than the nominal value. Simultaneously, the ultimate tensile strain value of the softening point was bound to be greater than or equal to the tensile strain value of the maximum stress point, according to the position relationship between the maximum stress point and the softening point. Tab. 2 indicates that the tensile strain ratio between the softening and maximum stress points of the five HDCC groups varies from 1.00 to 1.19. From these findings, the HDCC's uniaxial tension bilinear constitutive equation definitely provides a certain bearing capacity and tensile deformation redundancy for engineering structural design applications.

5 Conclusions

1) A simplified bilinear model of the constitutive relationship of HDCCs was established on the basis of the characteristics of the uniaxial tensile stress-strain curve of HDCCs.

2) The ideal elastoplastic linear strengthening model was the basis for the proposed HDCC bilinear tensile constitutive equation, which defines the control points for the nominal initial crack and nominal maximum tensile stress.

3) A comparative evaluation of the test and calculation results of five groups of HDCC experimental data depicted that the measured maximum stress values were larger than the nominal values, typically demonstrating that the ratios ranges from 1.08 to 1.22. Moreover, the ultimate tensile strain at the softening point was greater than or equal to the tensile strain at the maximum stress point, in the ratio range of 1.00 to 1.19.

4) Compared with the common constitutive equation, the application of the proposed uniaxial tension constitutive equation in the HDCC structural design calculations will yield a greater redundancy in the structural bearing capacity and tensile deformation.

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高延性水泥基复合材料单轴拉伸本构关系

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摘要:为了建立高延性水泥基复合材料(HDCC)在单轴拉伸荷载作用下的本构关系,指导 HDCC 的结构设计,在分析与总结现有单轴拉伸本构关系的基础上,基于理想弹塑性线性强化模型,以名义初裂点和名义最大应力点为控制点,提出了 HDCC 单轴拉伸双线性本构方程,结合 5 组 HDCC 单轴拉伸应力-应变曲线,进行了拉伸本构方程的应用示范与控制点关键参数的分析比较. 结果表明:所提出的本构方程克服了现有模型在计算承载力时富余度不足或过高的问题;实测最大应力值大于名义值,两者之比为 1.08 ~1.22;通过应变硬化阶段曲线的谷值点拟合直线所得到的软化点拉伸应变值大于或等于实测最大应力点的拉伸应变值,两者的比值为 1.00 ~1.19.

关键词:单轴拉伸;本构方程;简化模型;高延性水泥基复合材料

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