

Generalized nested logit-based stochastic user equilibrium model with distance constraint of electric vehicles

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Abstract: Considering the range anxiety issue caused by the limited driving range and the scarcity of battery charging stations, the conventional multinomial logit (MNL) model with the overlapping path issue was used in route choice modeling to describe the route choice behavior of travelers effectively. Furthermore, the generalized nested logit-based stochastic user equilibrium (GNL-SUE) model with the constraints of multiple user classes and distance limits was proposed. A mathematical model was developed and solved by the method of successive averages. The mathematical model was proven to be analytically equivalent to the modified GNL-SUE model, and the uniqueness of the solution was also confirmed. The proposed mathematical model was tested and compared with the GNL-SUE model without a distance limit and the MNL-SUE model with a distance limit. Results show that the proposed mathematical model can effectively handle the range anxiety and overlapping path challenges.

Key words: traffic engineering; stochastic user equilibrium; generalized nested logit; multinomial logit; method of successive averages; distance limit

DOI: 10.3969/j.issn.1003-7985.2022.02.011

Road traffic is one of the main sources of energy consumption and environmental pollution because vehicles on the road are dependent on oil-derived fuels in conventional gasoline vehicles (GVs). These fuels accelerate energy consumption and generate large amounts of tailpipe emissions of pollutants, such as NO_x and VOCs^[1]. Instead of gasoline-powered automobiles, automotive manufacturers and research institutes are striving to look for new types of vehicles, such as electric vehicles (EVs), to cope with these problems^[2]. Compared with conventional GV, EVs, especially battery EVs (BEVs), can significantly reduce greenhouse emissions and mitigate excessive consumption of fossil fuels^[3-5].

BEVs are also known to rely entirely on electricity. The electricity consumption of BEVs is generally proportional to the driving distance, and the driving range of BEVs is limited by the battery capacity. BEVs cannot be driven any farther longer once their driving distance is larger than their driving distance limitation because the battery storage of BEVs has been exhausted, and public electricity-charging stations of BEVs are insufficient^[6]. In this case, the fear of running out of batteries while traveling has led to the so-called range anxiety issue. Battery storage and charging technologies have experienced significant progress, and the number of new public charging stations is also increasing. However, range anxiety remains to be one of the main barriers hindering the widespread acceptance and adoption of BEVs^[7]. A general belief is that range anxiety might not be completely eliminated in the near future^[8-9]. Range anxiety will continue to affect the travel choices of BEV drivers. Therefore, the range anxiety issue must be properly addressed in the process of traffic assignment modeling, particularly for regions where BEVs are expected to grow rapidly.

Studies on incorporating range anxiety issues into the traffic assignment model have been conducted in the literature. Jiang et al.^[8] proposed a user equilibrium (UE) model with a distance limit, which is described by setting the path flow to zero when the path length exceeds the driving range limit. Jiang et al.^[10] further extended the model to investigate the coexistence of GV and BEVs, as well as their combined choices of destination, route, and parking, which are affected by the distance limit. Jiang et al.^[6] formulated a network equilibrium problem regarding the selection of modes and routes based on travelers who own GV and BEVs and analyzed the effects of operation costs and distance limits on vehicle and route choices of these travelers. However, these studies did not consider the behaviors of BEV users at the charging stations. To this end, He et al.^[9] and Xie et al.^[11] considered the time required for recharging and incorporated the relay requirement of EVs in their long-haul trips, which is typically beyond the distance limit. Xu et al.^[12] developed a mathematical model for the UE problems by considering battery swapping stations and road grade constraints. On this basis, Liu et al.^[13] presented the UE conditions to describe the route choice behaviors

Received 2021-11-21, **Revised** 2022-03-06.

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Foundation item: The Postgraduate Research & Practice Innovation Program of Jiangsu Province (No. KYLX16_0271).

Citation: Yan Dongmei, Guo Jianhua, Park B. Brian. Generalized nested logit-based stochastic user equilibrium model with distance constraint of electric vehicles [J]. Journal of Southeast University (English Edition), 2022, 38(2): 189 – 194. DOI: 10.3969/j.issn.1003-7985.2022.02.011.

of BEV drivers considering flow-dependent electricity consumption. Tran et al. [14] developed a bi-level optimization framework to determine the optimal location of public fast-charging stations. Notably, the above-mentioned traffic assignment models are mainly in the UE context. In this regard, Jing et al. [15] formulated a general MNL-based SUE model with a distance limit to extend the UE model. Riemann et al. [16] proposed a mixed-integer nonlinear model to determine the optimal location of wireless charging facilities for EVs. The model considers the interaction between the facility location and the traffic flow pattern captured by the MNL-based SUE model. Yang et al. [17] proposed a stochastic UE model for EVs by considering travel time, energy consumption, and charging service time. Huan et al. [18] proposed a dynamic traffic flow assignment model considering the en-route fast-charging behavior of users. Gao et al. [19] proposed a bilevel model to investigate the interaction between traffic flow distribution and the location of charging stations in a hybrid network of EVs and traditional GV.

Traffic assignment is a fundamental technique for transportation management. The stochastic user equilibrium (SUE)-based traffic assignment model has been widely adopted to deal with various perceptions of travelers. Most SUE models in the literature use the multinomial logit (MNL) route choice model. However, the assignment results are generally inadequate or impossible to account for similarities between different paths, that is, the overlapping path issue, due to the well-known IIA property of the MNL model [20]. Daganzo et al. [21] proposed the multinomial probit (MNP) model, which has an advantage that the similarity between alternatives can be considered. However, the MNP model is computationally unattractive because its probability function is not a closed form. In addition, other discrete choice models that can capture similarity among various routes have been extensively studied. The first category is modifications to MNL, such as C-logit [22–23] and path-size logit [24–25], which capture similarities through additional terms in the systematic utilities of the various routes. The second category is the generalized extreme value theory, such as the paired combinatorial logit (PCL) model [26–27], cross-nested logit (CNL) model [28–30], and generalized nested logit (GNL) model [31], which capture similarities by allowing additional general correlation structures. Notably, the PCL and CNL models are special cases of the GNL model. Furthermore, some researchers have adopted the GNL-based route choice model in the SUE modeling process. Bekhor et al. [31] proposed a mathematical formulation of the SUE model with GNL by adding an entropy term to the objective function. Li et al. [32] extended the model in Ref. [31] to formulate a GNL-based multiclass multicriteria SUE model. From a different perspective, Koppelman et al. [33] proposed a heterogeneous GNL model

that allows for heterogeneity in error variance and covariance structure to represent the complex behavioral processes involved in choice decision-making.

The range anxiety and overlapping path issues are essential or critical for traffic assignment with motor vehicles involving BEVs. Numerous studies were proposed to address the overlapping path and the range anxiety issues in traffic assignment. However, none of these models addressed the two issues together. Thus, this paper aims to develop and evaluate a framework addressing the two issues simultaneously. Hence, a generalized nested logit-based SUE (GNL-SUE) model with multiple user classes (i.e., GV and BEV users) and distance limits was designed. In addition, the analytical solution was proposed to solve this modified GNL-SUE model.

1 Generalized Nested Logit Model Review

The GNL model is described as a two-level nesting structure. Specifically, the upper layer comprises all links (nests) in the network, and the lower layer includes all paths (alternatives) in the path set. Assuming that this structure is adopted, the probability of choosing path k for user class i between O-D pair w is then given by

$$P_{ki}^w = \frac{\sum_m \left[(\alpha_{mk}^w \exp(-\theta_i c_{ki}^w))^{1/\mu_m^w} \left(\sum_{k \in K_m} (\alpha_{mk}^w \exp(-\theta_i c_{ki}^w))^{1/\mu_m^w} \right)^{\mu_m^w - 1} \right]}{\sum_m \left[\sum_{k \in K_m} (\alpha_{mk}^w \exp(-\theta_i c_{ki}^w))^{1/\mu_m^w} \right]^{\mu_m^w}} \quad \forall w, i, k \quad (1)$$

where c_{ki}^w is the travel cost of user class i on path k between OD pair w ; θ_i is the dispersion parameter that characterizes the familiarity of user class i with the road network; α_{mk}^w is the inclusion coefficient, which indicates that the portion of alternative k is assigned to nest m between O-D pair w . Prashker et al. [34] indicated that α_{mk}^w is computed as $\alpha_{mk}^w = (L_m^w / L_k^w)^\gamma \delta_{mk}^w$, where L_m^w is the length of link m between O-D pair w , L_k^w is the length of path k between O-D pair w , γ is a parameter that characterizes the driver's perception of the similarities between different routes, $\gamma = 1$, and δ_{mk}^w is a binary variable that indicates whether a path contains a link: δ_{mk}^w is equal to 1 if link m is on path k between O-D pair w and 0 otherwise; μ_m^w is the nesting coefficient for each nest m between O-D pair w solved as $\mu_m^w = 1 - \frac{1}{K_m} \sum_k \alpha_{mk}^w$, where K_m is the set of all alternatives included in nest m . The nesting coefficient μ_m^w satisfies $0 < \mu_m^w \leq 1$. In particular, $\mu_m^w = 1$ indicates the absence of a common link between the two paths, in which case the GNL model collapses to the MNL model.

Eq. (1) can be further decomposed into marginal and conditional probabilities, and the expression may be rewritten as

$$P_{ki}^w = \sum_m P_i^w(m) P_i^w(k | m) \quad \forall w, i, k \quad (2)$$

The marginal probability can be described as

$$P_i^w(m) = \frac{\left(\sum_{k \in K_m} (\alpha_{mk}^w \exp(-\theta_i c_{ki}^w))^{1/\mu_m^w} \right)^{\mu_m^w}}{\sum_m \left(\sum_{k \in K_m} (\alpha_{mk}^w \exp(-\theta_i c_{ki}^w))^{1/\mu_m^w} \right)^{\mu_m^w}} \quad \forall w, i, m \quad (3)$$

and the conditional probability may be given by the following equation:

$$P_i^w(k | m) = \frac{(\alpha_{mk}^w \exp(-\theta_i c_{ki}^w))^{1/\mu_m^w}}{\sum_{k \in K_m} (\alpha_{mk}^w \exp(-\theta_i c_{ki}^w))^{1/\mu_m^w}} \quad \forall w, i, m, k \quad (4)$$

2 Proposed Mathematical Model for the Modified GNL-SUE Model

2.1 Modified GNL-SUE model

The conventional GNL-SUE model is modified in this paper by incorporating the constraints of multiple user classes of GVs and BEVs and the distance limit of BEVs to handle the range anxiety and overlapping path issues jointly. To simplify the complexity of the modified GNL-SUE model, a series of assumptions are proposed:

- 1) The travel demand population includes only GV and BEV users.
- 2) The total travel demand for each type of vehicle at each origin is predetermined.
- 3) GV and BEV users have the same travel time on the same path.
- 4) Without loss of generality, all EVs are fully charged at their origins.

2.2 Proposed mathematical model

The objective function of the modified GNL-SUE model with multiple user classes and distance limits can be decomposed into UE mathematical formulation and two entropy terms. Specifically, the first term (Z_1) characterizes the deterministic UE formulation. The second term (Z_2) is similar to the MNL-SUE model in the formulation of Fisk^[35] but is modified to contain inclusion and nesting coefficients. The third term (Z_3) shows that the flow f_{mki}^w is aggregated by all paths. The modified GNL-SUE model with multiple user classes and distance limits can be expressed as the mathematical model shown below based on the above analyses:

$$\min Z = Z_1 + Z_2 + Z_3$$

where

$$\begin{aligned} Z_1 &= \sum_a \left(\int_0^x t_a(\omega) d\omega \right) \\ Z_2 &= \frac{1}{\theta_i} \sum_w \sum_i \sum_m \sum_k \mu_m^w f_{mki}^w \ln \frac{f_{mki}^w}{(\alpha_{mk}^w)^{1/\mu_m^w}} \\ Z_3 &= \frac{1}{\theta_i} \sum_w \sum_i \sum_m (1 - \mu_m^w) \left(\sum_k f_{mki}^w \right) \ln \left(\sum_k f_{mki}^w \right) \end{aligned} \quad (5)$$

s. t.

$$\sum_m \sum_k f_{mki}^w = q_i^w \quad \forall w, i \quad (6)$$

$$(D_i - L_k^w) f_{mki}^w \geq 0 \quad \forall w, i, m, k \quad (7)$$

$$x_{ai} = \sum_w \sum_m \sum_k f_{mki}^w \delta_{ak}^w \quad \forall a, i \quad (8)$$

$$x_a = \sum_i x_{ai} \quad \forall a \quad (9)$$

$$f_{mki}^w \geq 0 \quad \forall w, i, m, k \quad (10)$$

where f_{mki}^w is the flow on path k belonging to nest m for user class i between O-D pair w ; q_i^w is the total demand of user class i between O-D pair w ; x_a is the flow on link a ; x_{ai} is the flow on link a for user class i ; D_i is the distance limit of user class i ; L_k^w is the length of path k between O-D pair w .

Eq. (6) represents the flow conservation constraints; that is, the flow on all paths connecting each O-D pair must be equal to the O-D trip demand for user class i . Eq. (7) indicates that if the path length is less than or equal to the distance limit for a given user class i , then the flow on that path is positive; otherwise, the path flow should be equal to zero. Eq. (8) denotes the incidence relationship between link-path flows. Eq. (9) is a summation of the link flows of the user class i , while Eq. (10) is the flow of nonnegativity constraint.

2.3 Equivalence and uniqueness conditions

Proposition 1 The proposed mathematical model is equivalent to the modified GNL-SUE model with multiple user classes and distance limits.

Proof Any flow pattern obtained by solving Eqs. (5) to (10) is generally acknowledged to satisfy the SUE condition. The Lagrangian of the proposed mathematical model can be formulated as

$$\begin{aligned} L(f, \mu, \lambda) &= Z + \sum_w \sum_i \tau_i^w \left(q_i^w - \sum_m \sum_k f_{mki}^w \right) - \\ &\quad \sum_w \sum_i \sum_m \sum_k \lambda_{mki}^w (D_i - L_k^w) f_{mki}^w \end{aligned} \quad (11)$$

where τ_i^w and λ_{mki}^w are dual variables associated with the flow conservation constraint.

The first-order conditions of the proposed mathematical model are equivalent to the first-order conditions of Lagrangian. Therefore, the following conditions must hold at the stationary point of the Lagrangian considering the path-flow variable, that is,

$$f_{mki}^w \frac{\partial L(f, \mu, \lambda)}{\partial f_{mki}^w} = 0, \quad \frac{\partial L(f, \mu, \lambda)}{\partial f_{mki}^w} \geq 0 \quad \forall w, i, m, k \quad (12)$$

The partial derivatives of $L(f, \mu, \lambda)$ considering the path-flow variable is then given by

$$\frac{\partial L(f, \mu, \lambda)}{\partial f_{mki}^w} = \sum_a t_a(x_a) \delta_{ak}^w + \frac{\mu_m^w}{\theta_i} \ln \frac{f_{mki}^w}{(\alpha_{mk}^w)^{1/\mu_m^w}} + \frac{\mu_m^w}{\theta_i} +$$

$$\frac{1 - \mu_m^w}{\theta_i} \ln \left(\sum_k f_{mki}^w \right) + \frac{1 - \mu_m^w}{\theta_i} - \tau_i^w - \lambda_{mki}^w (D_i - L_k^w) \quad (13)$$

$L(f, \mu, \lambda)$ must be minimized considering nonnegative path flows. Thus, the expression can be rewritten as

$$\sum_a t_a(x_a) \delta_{ak}^w + \frac{\mu_m^w}{\theta_i} \ln \frac{f_{mki}^w}{(\alpha_{mk}^w)^{1/\mu_m^w}} + \frac{\mu_m^w}{\theta_i} + \frac{1 - \mu_m^w}{\theta_i} \ln \left(\sum_k f_{mki}^w \right) + \frac{1 - \mu_m^w}{\theta_i} - \tau_i^w - \lambda_{mki}^w (D_i - L_k^w) = 0 \quad (14)$$

Assume that the composite path cost for user class i includes the path travel time and $\lambda_{mki}^w (L_k^w - D_i)$. In particular, $\lambda_{mki}^w (L_k^w - D_i)$ is the path out-of-range cost incurred when the path length exceeds the distance limit of the user class i . The equation of path cost is defined as

$$H_{mki}^w = \sum_a t_a(x_a) \delta_{ak}^w + \lambda_{mki}^w (L_k^w - D_i) \quad (15)$$

Submitting Eq. (15) into Eq. (14) and multiplying both sides by θ_i , the following is obtained:

$$\mu_m^w \ln \frac{f_{mki}^w}{(\alpha_{mk}^w)^{1/\mu_m^w}} + (1 - \mu_m^w) \ln \left(\sum_k f_{mki}^w \right) + \theta_i H_{mki}^w + 1 - \theta_i \tau_i^w = 0 \quad (16)$$

Dividing both sides by μ_m^w and taking the logarithm of exp as the base, Eq. (16) can be rewritten as

$$\frac{f_{mki}^w}{\left(\sum_k f_{mki}^w \right)^{(1 - \mu_m^w)/\mu_m^w}} = \exp \left(\frac{\theta_i \tau_i^w - 1}{\mu_m^w} \right) (\alpha_{mk}^w)^{1/\mu_m^w} \exp \left(\frac{-\theta_i H_{mki}^w}{\mu_m^w} \right) \quad (17)$$

Summing the above expression by path k , the following is obtained:

$$\left(\sum_k f_{mki}^w \right)^{1/\mu_m^w} = \exp \left(\frac{\theta_i \tau_i^w - 1}{\mu_m^w} \right) \sum_k (\alpha_{mk}^w)^{1/\mu_m^w} \exp \left(\frac{-\theta_i H_{mki}^w}{\mu_m^w} \right) \quad (18)$$

Elevating both sides to μ_m^w ,

$$\sum_k f_{mki}^w = [\exp(\theta_i \tau_i^w - 1)] \left[\sum_k (\alpha_{mk}^w)^{1/\mu_m^w} \cdot \exp \left(\frac{-\theta_i H_{mki}^w}{\mu_m^w} \right) \right]^{\mu_m^w} \quad (19)$$

Summing Eq. (19) by link (nest) m ,

$$\sum_m \sum_k f_{mki}^w = [\exp(\theta_i \tau_i^w - 1)] \sum_m \left[\sum_k (\alpha_{mk}^w)^{1/\mu_m^w} \cdot \exp \left(\frac{-\theta_i H_{mki}^w}{\mu_m^w} \right) \right]^{\mu_m^w} \quad (20)$$

Dividing Eq. (19) by Eq. (20), the marginal probability that nest m will be chosen is obtained as shown below.

$$P_i(m) = \frac{\sum_k f_{mki}^w}{\sum_m \sum_k f_{mki}^w} =$$

$$\frac{\left\{ \sum_k (\alpha_{mk}^w)^{1/\mu_m^w} \exp \left[(-\theta_i H_{mki}^w)/\mu_m^w \right] \right\}^{\mu_m^w}}{\sum_m \left\{ \sum_k (\alpha_{mk}^w)^{1/\mu_m^w} \exp \left[(-\theta_i H_{mki}^w)/\mu_m^w \right] \right\}^{\mu_m^w}} \quad (21)$$

Dividing Eq. (18) by Eq. (19),

$$P_i(k | m) = \frac{f_{mki}^w}{\sum_k f_{mki}^w} = \frac{(\alpha_{mk}^w)^{1/\mu_m^w} \exp \left[(-\theta_i H_{mki}^w)/\mu_m^w \right]}{\sum_k (\alpha_{mk}^w)^{1/\mu_m^w} \exp \left[(-\theta_i H_{mki}^w)/\mu_m^w \right]} \quad (22)$$

Thus, the conditional probability of path k will be chosen in nest m .

Therefore, Eqs. (21) and (22) correspond to Eqs. (3) and (4), respectively. Notably, the path costs in Eqs. (1), (3), and (4) are denoted by H_{mki}^w because the path out-of-range cost when the path length exceeds the distance limit is included. Thus, the mathematical formulation presented in Eq. (5) corresponds to the GNL-SUE model with a distance limit.

Proposition 2 The proposed mathematical model has a unique solution.

Proof Demonstrating that the Hessian matrix of the objective function of the proposed mathematical model is positive definite is sufficient to prove the uniqueness of the solution. The feasible region of Z_1 is the same as Fisk's formulation; thus, it is convex. The second derivative of Z_2 and Z_3 can be calculated as follows:

$$\frac{\partial^2 Z_2}{\partial f_{mki}^w \partial f_{mli}^w} = \begin{cases} \frac{\mu_m^w}{\theta_i f_{mki}^w} & k = l \\ 0 & \text{otherwise} \end{cases} \quad (23)$$

$$\frac{\partial^2 Z_3}{\partial f_{mki}^w \partial f_{mli}^w} = \frac{1 - \mu_m^w}{\theta_i \sum_k f_{mki}^w} \quad (24)$$

Obviously, the Hessian matrices of Z_2 and Z_3 are positive definite and positive semidefinite, respectively, which ensures the convexity of the objective function. Thus, the solution to the proposed mathematical model is unique considering the path-flow variable f_{mki}^w .

2.4 MSA-based solution to the proposed mathematical model

The method of successive average (MSA) is adopted to solve the proposed mathematical model. The application steps of the MSA algorithm to solve the proposed mathematical model are presented below.

Step 1 Parameter settings. Determine a feasible path set. Meanwhile, set the distance limit D and the convergence tolerance ε .

Step 2 Initialization. Calculate inclusion coefficients α and nesting coefficients μ . Find an initial path flow $f^{(1)}$. Set iteration counter $n = 1$.

Step 3 Update. Calculate the link flow $x^{(n)}$ based on the current path flow $f^{(n)}$. Update the link travel time

$c(\mathbf{x}^{(n)})$ and path travel time $T^{(n)}$.

Step 4 Direction finding. Perform a stochastic network loading based on the current set of path travel time $T^{(n)}$, yielding an auxiliary solution $\mathbf{Y}^{(n)}$.

Step 5 Move. Set $\mathbf{f}^{(n+1)} = \mathbf{f}^{(n)} + (1/n) (\mathbf{Y}^{(n)} - \mathbf{f}^{(n)})$. Notably, the step size is the reciprocal of the number of iterations.

Step 6 Convergence test. If $\|\mathbf{f}^{(n+1)} - \mathbf{f}^{(n)}\| / \|\mathbf{f}^{(n)}\| \leq \varepsilon$, then stop; else, set $n = n + 1$ and go to Step 3.

3 Numerical Examples

Two road networks are selected in this section for testing the purpose of this paper. Specifically, a small network containing overlapping and nonoverlapping paths is first selected to show the performance of the proposed mathematical model handling the overlapping path issue. The result indicates that the distance limit has an important impact on the path and link flows at equilibrium. The well-known Nguyen-Dupuis network is selected to demonstrate the performance of the proposed mathematical model handling the range anxiety issue by analyzing the effect of changes in distance limits on the path flows of GV and BEV.

3.1 Small network: Performance for overlapping path issue

The small network comprises 1 O-D pair, 5 links, and 3 paths, as shown in Fig. 1. The total demand is equal to 1000. In addition, the BPR function is used to calculate link travel time, in which its coefficients are set to $\alpha = 0.15$ and $\beta = 4$. The distance limit of BEVs is set to 10 herein, and the distance limit of GVs is unlimited. The market share for GVs and BEVs is defined as 0.5 and 0.5, respectively. The dispersion parameters of GV and BEV users are respectively set to 0.3 and 0.5. The iteration accuracy of the MSA-based solution algorithm is defined as $\varepsilon = 0.0001$. The free-flow travel time and capacity for each link on this small network are shown in Tab. 1, and the path composition and length are exhibited in Tab. 2. The values in Tabs. 1 and 2 are adopted from Li et al^[32].

The path and link flows calculated by the proposed mathematical model (i.e., the GNL-SUE model with a distance limit), the GNL-SUE model without a distance limit, and the MNL-SUE model with a distance limit are also shown in Figs. 2 (a), (b), and (c), respectively.

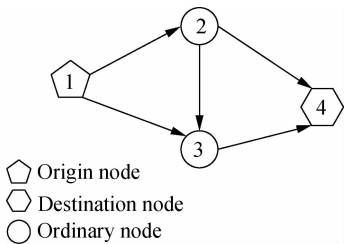


Fig. 1 Small network

Tab. 1 Link characteristics of the small network

No.	Link	t_a^0	\bar{c}_a
1	1-2	5	600
2	1-3	6	500
3	2-3	7	600
4	2-4	8	500
5	3-4	3	700

Tab. 2 Path composition and path length

O-D	Path	Node sequence	Path length
1-4	1	1-2-4	13
	2	1-2-3-4	15
	3	1-3-4	9

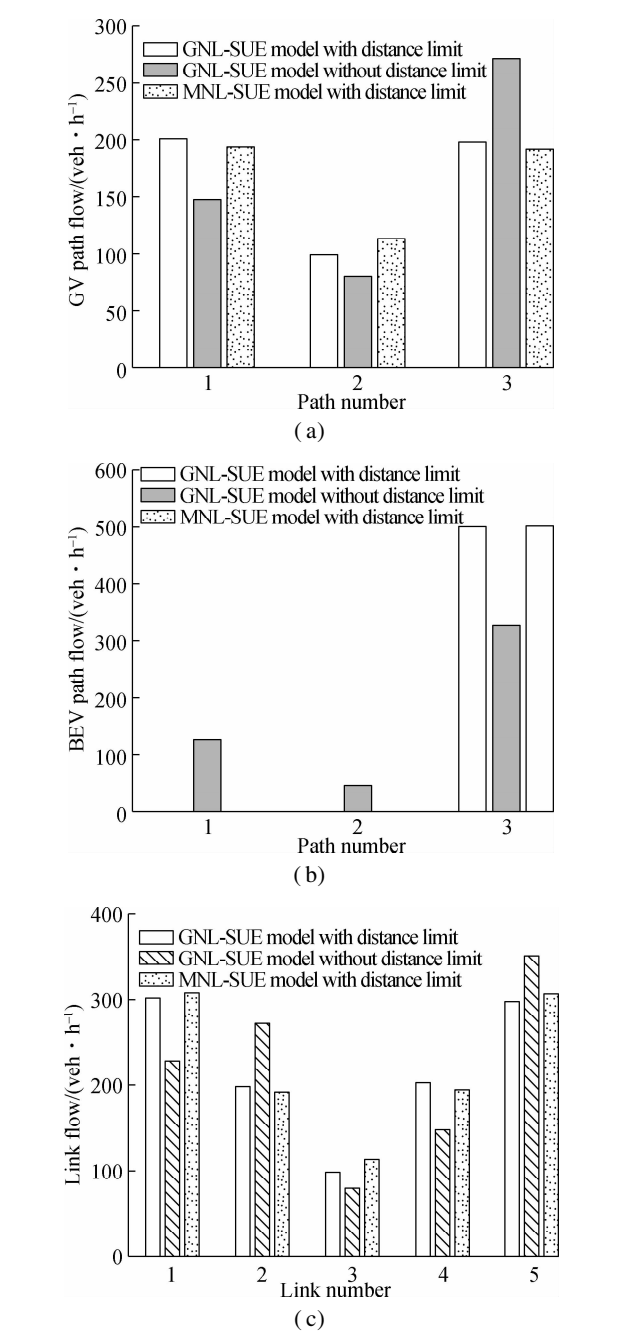


Fig. 2 Comparison of path flows and link flows at equilibrium. (a) Comparison of GV path flows; (b) Comparison of BEV path flows; (c) Comparison of link flows

Fig. 2 shows that the path and link flows assigned by the proposed mathematical model are different from the GNL-SUE model without a distance limit and the MNL-SUE model with a distance limit. This difference is due to the consideration of the distance limit of the BEVs and the similarities between paths by the proposed model. More specifically, Figs. 2(a) and (b) reveal the results of path flows assigned by the proposed mathematical model and the GNL-SUE model without a distance limit, wherein GV flow is assigned to paths 1, 2, and 3, while BEV flow is not assigned to paths 1 and 2. GV users are unaffected by the distance limit, while the length of paths 1 and 2 are greater than the distance limit of BEVs. Therefore, no BEV users travel on the two paths.

Additionally, Fig. 2(c) shows that the link flows assigned by the proposed mathematical model on links 1 and 5 are smaller than those of the MNL-SUE model with a distance limit. Meanwhile, the link flows assigned by the proposed mathematical model on links 2 and 4 are larger than those of the MNL-SUE model with a distance limit. This finding is due to the MNL-SUE model with a distance limit, which does not consider the similarities between the paths. Therefore, the flows of the overlapping links are overestimated. By contrast, the proposed mathematical model overcomes the IIA drawback of the MNL-SUE model with a distance limit by considering the similarity among various paths. Therefore, the links with multiple overlapping paths are assigned fewer flows than the MNL-SUE model with a distance limit.

3.2 Nguyen-Dupuisnet work: Performance for range anxiety issues

The Nguyen-Dupuis network is selected to analyze the effect of the distance limit parameter on the path flows of GVs and BEVs. This network includes 4 O-D pairs, 13 nodes, 19 links, and 25 paths, as shown in Fig. 3. The total demand for each O-D pair is $q_{1-2} = 660$, $q_{1-3} = 495$, $q_{4-2} = 412.5$, and $q_{4-3} = 495$. Other corresponding parameters are the same as those set in the small network. The free-flow travel time and capacity for each link on the Nguyen-Dupuis network are shown in Tab. 3. The values in Tab. 3 are adopted from Xu et al^[36]. The path composition and length are exhibited in Tab. 4. The values in Tab. 4 are adopted from Jiang et al^[6].

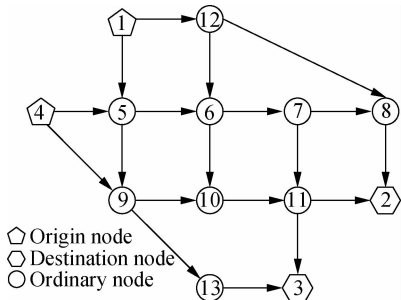


Fig. 3 Nguyen-Dupuis network

Tab. 3 Link characteristics

No.	Link	t_a^0	\bar{c}_a	No.	Link	t_a^0	\bar{c}_a
1	1-5	7	300	11	8-2	9	500
2	1-12	9	200	12	9-10	10	550
3	4-5	9	200	13	9-13	9	200
4	4-9	12	200	14	10-11	6	400
5	5-6	3	350	15	11-2	9	300
6	5-9	9	400	16	11-3	8	300
7	6-7	5	500	17	12-6	7	200
8	6-10	13	250	18	12-8	14	300
9	7-8	5	250	19	13-3	11	200
10	7-11	9	300				

Tab. 4 Path composition and path length

O-D	Path	Node sequence	Path length
1-2	1	1-12-8-2	32
	2	1-5-6-7-8-2	29
	3	1-5-6-7-11-2	33
	4	1-5-6-10-11-2	38
	5	1-5-9-10-11-2	41
	6	1-12-6-7-8-2	35
	7	1-12-6-7-11-2	39
	8	1-12-6-10-11-2	44
1-3	9	1-5-9-13-3	36
	10	1-5-6-7-11-3	32
	11	1-5-6-10-11-3	37
	12	1-5-9-10-11-3	40
	13	1-12-6-7-11-3	38
	14	1-12-6-10-11-3	43
4-2	15	4-9-10-11-2	37
	16	4-5-6-7-8-2	31
	17	4-5-6-7-11-2	35
	18	4-5-6-10-11-2	40
	19	4-5-9-10-11-2	43
	25	4-5-9-10-11-3	42
4-3	20	4-9-13-3	32
	21	4-9-10-11-3	36
	22	4-5-9-13-3	38
	23	4-5-6-7-11-3	34
	24	4-5-6-10-11-3	39

The variation of the path flows of GV and BEV with distance limits is depicted in Fig. 4 to examine the impacts of the distance limit on the path flows of GV and BEV in each O-D pair.

Fig. 4 shows that only 2 to 4 paths in each OD pair are available and carry BEV flows when D tightens. BEV flows are assigned to additional paths due to the increase in the number of paths available when D loosens, and the BEV flows gradually tend to stabilize. By contrast, the GV flows are assigned to each path in four O-D pairs because no distance limit exists. Moreover, the GV and BEV flows on some paths (e. g., paths 1, 10, 16, and 20) show opposite variations as D increases. This phenomenon may be explained by the strong influence of distance limits on the route choice behavior of BEV users. In other words, BEV users are not allowed to travel on

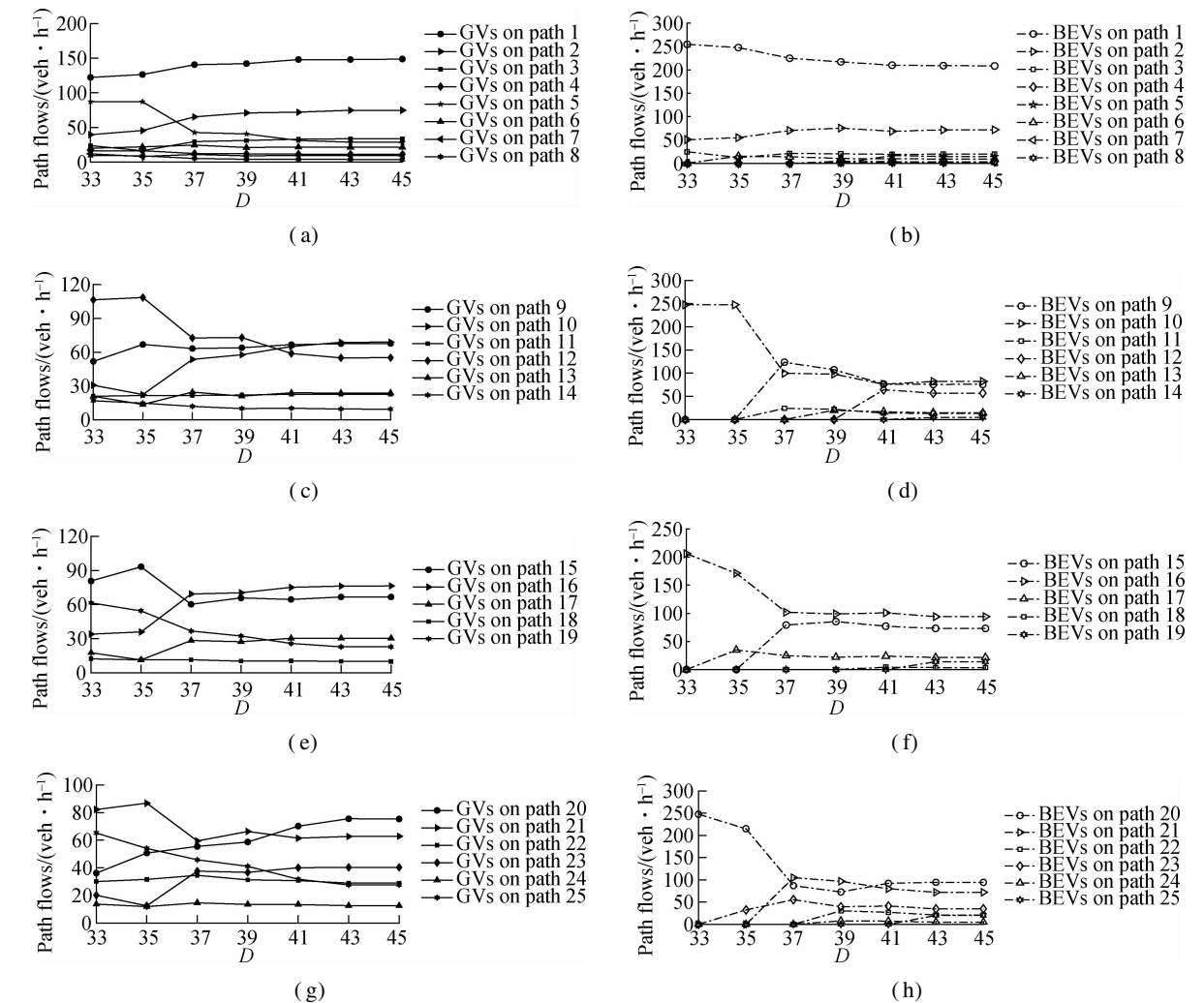


Fig. 4 Variation of path flows of GV and BEV with the distance limit. (a) GV path flows in O-D pair (1, 2); (b) BEV path flows in O-D pair (1, 2); (c) GV path flows in O-D pair (1, 3); (d) BEV path flows in O-D pair (1, 3); (e) GV path flows in O-D pair (4, 2); (f) BEV path flows in O-D pair (4, 2); (g) GV path flows in O-D pair (4, 3); (h) BEV path flows in O-D pair (4, 3)

paths restricted by distance; thus, they tend to choose short paths to complete their trips. These short paths will then become oversaturated due to the entry of BEV users, which causes a significant decline in the capacity of the links contained in those paths and a substantial increase in path travel time. Therefore, GV users prefer to use these unsaturated paths to reduce their travel time.

4 Conclusions

- 1) A modified GNL-SUE model is proposed by incorporating the constraints of multiple user classes with distance limits on BEVs. An equivalent mathematical model and associated solutions were also provided for the proposed model.
- 2) The distance limit has an impact on the path and link flows at equilibrium, and the proposed mathematical model can overcome the IIA drawback of the MNL-SUE model with distance limits by handling the overlapping path issue.
- 3) Results using the Nguyen-Dupuis network show that the proposed approach can handle the range anxiety issue

by selecting different distance limit parameters in traffic assignment.

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电车距离约束条件下的广义巢式 Logit 随机用户平衡模型

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摘要:为了更好地描述出行者的路径选择行为,路径选择建模过程中考虑了有限的驾驶范围和稀缺的充电站引起的范围焦虑问题,以及传统的多项式 Logit 模型具有的路径重叠问题.提出了多用户类和距离限制约束的广义巢式 Logit 随机用户平衡(GNL-SUE)模型,给出了相应的数学模型,并使用相继平均法(MSA)对其求解.证明了该数学模型等价于改进的 GNL-SUE 模型,以及模型解的唯一性.最后,测试了所提出的数学模型的性能,并将其与不含距离限制约束的 GNL-SUE 模型和含有距离限制约束的多项式 Logit 随机用户平衡(MNL-SUE)模型进行比较.结果表明,所提出的数学模型能够解决范围焦虑问题和路径重叠问题.

关键词:交通工程;随机用户平衡;广义巢式 Logit;多项式 Logit;相继平均法;距离限制

中图分类号:U491