A modified time domain interpolation method for LS channel estimation in OFDM systems

Zhang Yenan　　Xu Feiyun　　Jia Minping

(School of Mechanical Engineering, Southeast University, Nanjing 211189, China)

Abstract: A modified time domain interpolation method is proposed for orthogonal frequency division multiplexing (OFDM) systems to address the problem that time domain interpolation in the least square (LS) channel estimation method based on comb-type pilots cannot choose the pilot spacing flexibly. Firstly, the estimated channel frequency response (CFR) at pilot positions in the frequency domain is obtained by LS channel estimation based on comb-type pilots, and the estimated channel impulse response (CIR) in the time domain is obtained by linear interpolation and inverse fast Fourier transform (IFFT). Secondly, the error of the estimated CIR obtained by linear interpolation is analyzed by theoretical deduction, and a method for correcting it is proposed. Finally, an estimated CFR at all subcarrier positions in the frequency domain is obtained by performing zero padding in the time domain and fast Fourier transform (FFT) on the modified CIR. The simulation results suggest that the proposed method gives similar performance to time domain interpolation, yet it does not need to meet the condition of time domain interpolation that the number of subcarriers must be an integral multiple of pilot spacing to use it. The proposed method allows for flexible pilot spacing, reducing the number of pilots and the consumption of subcarriers used for channel estimation.

Key words: orthogonal frequency division multiplexing (OFDM); channel estimation; linear interpolation; time domain interpolation

DOI: 10. 3969/j. issn. 1003 − 7985. 2022. 03. 002

Orthogonal frequency division multiplexing (OFDM) is a multicarrier modulation technique with high bandwidth efficiency, high transmission rate, and robustness to multipath fading. It has been adopted in wireline applications such as digital audio broadcasting (DAB)[1], underwater acoustic communication (UWA)[2], and digital video broadcasting (DVB)[3], fifth generation (5G)[4], etc.

Channel estimation is essential for OFDM systems; the influence of the channel needs to be estimated at the receiver. Pilot-based approaches are widely used in channel estimation. Pilot tones can be inserted into the time domain or frequency domain, called block-type and comb-type. The comb-type pilot arrangement has better performance than the block-type for fast-fading channels[5]. The estimated channel frequency response (CFR) at the pilot positions can be obtained by least square (LS) or minimum mean square error (MMSE)[6]. MMSE algorithm provides better performance with high complexity. LS algorithm utilizes the least-squares criterion and has been widely used due to its low complexity.

The LS estimated CFR is interpolated to obtain CFR at all subcarriers. The interpolation methods can be divided into frequency domain interpolation, transform domain interpolation, and time domain interpolation. The frequency domain interpolation methods include linear interpolation, Gaussian interpolation, and spline cubic interpolation[7]. Transform domain interpolation uses the discrete Fourier transform (DFT) to transform the LS estimated CFR into the transform domain. High-resolution interpolation with low complexity is based on zero-padding in the transform domain and inverse DFT (IDFT)[8].

Time domain interpolation has a similar process to transform domain interpolation. The LS estimated CFR is transformed into the time domain by IDFT, and the estimated channel impulse response (CIR) is obtained. CFR at all subcarriers is obtained by zero padding and DFT. Time domain interpolation has been proven to have a lower bit error rate (BER) than linear interpolation[7]. Time domain interpolation has a better performance than transform domain interpolation, as shown in Ref. [8]. Furthermore, as demonstrated in Ref. [9], time domain interpolation outperforms linear, cubic, and low pass interpolation. Time domain interpolation has been investigated for OFDM systems with virtual subcarriers in Refs. [10, 11].

Time domain interpolation is accurate if the channel delays are integral multiples of the sample time of the OFDM symbol and the sample length of CIR is less than the number of pilots[12]. Furthermore, DFT and IDFT are easy to realize using the fast Fourier transform (FFT) algorithm[13]. Thus, time domain interpolation is widely

used. Noise has the potential to affect LS, and noise reduction in the time domain is widely used[14−16]. Time domain interpolation is realized by zero-padding in the time domain. Therefore, it can be used with various time domain noise reduction methods.

However, pilot subcarrier spacing is determined by the channel's maximum excess delay[11]. The pilot subcarriers used in channel estimation will consume a significant amount of subcarrier resources for the channel with a large delay spread[17]. To use time domain interpolation, the number of subcarriers must be an integral multiple of the pilot subcarrier spacing[18], which may cause a larger consumption of pilots. A reasonable pilot spacing should ensure good channel estimation performance while minimizing the number of subcarriers consumed by the pilots[19]. The subcarriers' distribution is essential, especially for applications with narrow channel bandwidth[20−21].

In this paper, the modified time domain interpolation method is proposed to address the disadvantage of time domain interpolation in which the number of subcarriers must be an integral multiple of the pilot subcarrier spacing. Firstly, the OFDM system model is presented and the relationship between the estimated CIR from LS based on linear interpolation and the actual CIR is analyzed. Secondly, the disadvantages of time domain interpolation are examined and a modified time domain interpolation method is proposed. Then, simulation results are presented and analyzed.

1 Analysis of LS Channel Estimation Based on Linear Interpolation

1.1 System model

We consider an OFDM system that has N subcarriers. The transmitted signal at each subcarrier is $\{X[k],\ k=0,$ $1,\ \ldots,\ N-1\}$, where k represents the subcarrier index. The transmitted signal is transformed to a discrete-time domain by IDFT, given by

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \exp\left(j \frac{2\pi nk}{N}\right) \qquad 0 \leqslant n \leqslant N-1 \tag{1}$$

where n is the sample index of the OFDM signal in the time domain. This is the modulation process of the OFDM system. Considering the effect of multipath effects, discrete-time CIR is given by

$$h[n] = \sum_{l=0}^{L-1} \alpha_l \delta[n - \tau_l] \tag{2}$$

where l is the index of the different paths; α_l is the path complex gain of the l-th path; τ_l is the delay of the l-th path; and L is the total number of channel paths. If reasonable CP guard samples are used after passing through a multipath channel and removing CP, the received OFDM signal is represented by

$$r[n] = x[n] \otimes h[n] + z[n] \tag{3}$$

where $\otimes$ denotes an N-point circular convolution operation, and $z[n]$ denotes time domain samples of independent and identical distributed additive white Gaussian noise (AWGN). Assuming perfect synchronization, the k-th subcarrier symbol in the frequency domain is given by

$$R[k] = \mathrm{DFT}_N\{r[n]\} = X[k] H[k] + Z[k] \tag{4}$$

where $Z[k]$ are AWGN samples in frequency domain. $Z[k] = \mathrm{DFT}_N\{z[n]\}$, $H[k] = \mathrm{DFT}_N\{h[n]\}$, and $\mathrm{DFT}_N\{\}$ represents N-point DFT.

By inserting pilot tones at some subcarriers, CFR at pilot positions can be obtained by

$$\hat{H}[k] = \frac{R[k]}{X[k]} \tag{5}$$

This method is called LS estimation.

1.2 Analysis of linear interpolation

We consider LS estimation using comb-type pilots and linear interpolation. Pilot tones are uniformly inserted at an m interval. When N/m is an integer and the first pilot tone is inserted at the first subcarrier, the number of pilots $K = N/m$. The estimated CFR at pilot positions is $\{\hat{H}[im],\ i=0,\ 1,\ \ldots,\ K-1\}$, and $\hat{H}[N]$ is defined as

$$\hat{H}[N] \triangleq 2\,\hat{H}[N-m] - \hat{H}[N-2m] \tag{6}$$

Defining $\{\hat{H}_P[i] = \hat{H}[im],\ i=0,\ 1,\ \ldots,\ K\}$, by linear interpolation, the CFR at all subcarriers positions is given as

$$\hat{H}[k] = \hat{H}[im+p] = \left(1 - \frac{p}{m}\right)\hat{H}_P[i] + \frac{p}{m}\hat{H}_P[i+1]$$
$$p=0,1,\ldots,m-1;\ k=0,1,\ldots,N-1;\ i=0,1\ldots,K-1 \tag{7}$$

Further, the estimated CIR $\hat{h}_L[n]$ can be expressed by the N-point FFT of the estimated CFR, given by

$$\hat{h}_L[n] = \frac{1}{N} \sum_{k=0}^{N-1} \hat{H}[k] W_N^{-nk} = \frac{1}{N} \sum_{i=0}^{K-1} \sum_{p=0}^{m-1} \hat{H}[im+p] W_N^{-n(im+p)} \tag{8}$$

where $W_N = \exp(-j2\pi/N)$, called the rotation factor. By the properties of the rotation factor[22], we have

$$\hat{h}_L[n] = \frac{1}{N} \sum_{i=0}^{K-1} \sum_{p=0}^{m-1} \hat{H}[im+p] W_N^{-n(im+p)} = \sum_{p=0}^{m-1} W_N^{-np} \hat{h}[n,p] \tag{9}$$

when $0 \leqslant n \leqslant K-1$,

$$\hat{h}[n,p] = \frac{1}{N} \sum_{i=0}^{K-1} \hat{H}[im+p] W_K^{-ni} = \frac{(m-p)\,\hat{h}_P[n]}{m^2} +$$

$$\frac{pW_K^n\left(\hat{h}_P[n] + \dfrac{m}{N}(2\hat{H}_P[K-1] - \hat{H}_P[K-2] - \hat{H}_P[0])\right)}{m^2}$$

$$(10)$$

where $\hat{h}_P[n] = \mathrm{IDFT}_K\{\hat{H}_P[i], \ i = 0, 1, \ldots, K-1\}$.

By Eqs. (9) and (10), we have

$$\hat{h}_L[n] = P_{N,m}[n]\,\hat{h}_P[n] + \frac{1}{mN}\Delta H \sum_{p=0}^{m-1} pW_N^{(m-p)n} \quad (11)$$

where $P_{N,m}[n]$ is defined by

$$P_{N,m} \triangleq [n]\frac{1}{m^2}\left[\frac{\sin(\pi mn/N)}{\sin(\pi n/N)}\right]^2 \quad (12)$$

The condition that N/m is not an integer is then considered, and the other conditions are the same. The number of pilots $K = \langle N/m \rangle$, where $\langle N/m \rangle$ represents the nearest integer with a value greater than N/m.

As in Eq. (6), $\hat{H}[mK]$ is defined by

$$\hat{H}[mK] \triangleq 2\hat{H}[m(K-1)] - \hat{H}[m(K-2)] \quad (13)$$

$\{\hat{H}[k], \ k=0, 1, \ldots, mK-1\}$ is obtained by linear interpolation. Assuming $mK/N \approx 1$ and $W_N \approx W_{mK}$, by similar deduction to Eqs. (8) to (12), $\hat{h}_L[n]$ is represented by

$$\hat{h}_L[n] = \frac{1}{N}\sum_{k=0}^{N-1}\hat{H}[k]W_N^{-nk} \approx P_{mK,m}[n]\,\hat{h}_P[n] +$$

$$\frac{1}{m^2 K}(2\hat{H}_P[K-1] - \hat{H}_P[K-2] - \hat{H}_P[0])\sum_{p=0}^{m-1}pW_{mK}^{(m-p)n} -$$

$$\frac{1}{N}\sum_{k=N}^{mK-1}\hat{H}[k]W_N^{-nk} \triangleq P_{mK,m}[n]\,\hat{h}_P[n] + I[n] \quad (14)$$

When N is large, the last two parts of Eq. (14) are insignificant. Then we have

$$\hat{h}_L[n] \approx p_{mK,m}[n]\hat{h}_P[n] \qquad 0 \leqslant n \leqslant K-1 \quad (15)$$

2 Proposed Interpolation Method

2.1 Analysis of time domain interpolation

Time domain interpolation outperforms linear interpolation with low complexity in bit error rate (BER). It is realized by zero-padding and FFT/IFFT. The LS estimated CFR $\{\hat{H}_P[i], \ i = 0, 1, \ldots, K-1\}$ is converted into the time domain by IFFT:

$$\hat{h}_P[n] = \frac{1}{K}\sum_{i=0}^{K-1}\hat{H}_P[i]W_K^{-ni} \qquad 0 \leqslant n \leqslant K-1 \quad (16)$$

The K-sample time domain sequence $\hat{h}_P[n]$ is extended to an N-sample sequence $\hat{h}_{\mathrm{DFT}}[n]$ by $(N-K)$-point zero-padding. There are two basic methods of zero-padding: The first one is padding at the end of the sequence[8], given by

$$\hat{h}_{\mathrm{DFT}}[n] = \begin{cases} \hat{h}_P[n] & 0 \leqslant n \leqslant K-1 \\ 0 & K \leqslant n \leqslant N-1 \end{cases} \quad (17)$$

The second one is padding at the middle of the sequence[23], given by

$$\hat{h}_{\mathrm{DFT}}[n] = \begin{cases} \hat{h}_P[n] & 0 \leqslant n \leqslant \dfrac{K}{2}-1 \\ 0 & \dfrac{K}{2} \leqslant n \leqslant N - \dfrac{K}{2}-1 \\ \hat{h}_P[n-N+K+1] & N-\dfrac{K}{2} \leqslant n \leqslant N-1 \end{cases} \quad (18)$$

The first one is studied in this paper.

The estimated CFR at all subcarriers is obtained by N-point FFT:

$$\hat{H}_{\mathrm{DFT}}[k] = \sum_{n=0}^{N-1}\hat{h}_{\mathrm{DFT}}[n]W_N^{nk} \qquad k = 0, 1, \ldots, N-1 \quad (19)$$

To analyze the interpolation method separately, the LS estimated CFR is assumed to be accurate, i.e., $\hat{H}_P[i] = H[im]$. Considering the condition that the delay τ_l in Eq. (2) is an integer, and the maximum delay $\tau_m \leqslant K$, then $\{\hat{h}_P[n], \ 0 \leqslant n \leqslant K-1\}$ is the accurate estimated result of CIR[12], proved as follows:

$$\hat{H}_P[i] = H[im] = \sum_{n=0}^{N-1}h[n]W_N^{inm} = \sum_{n=0}^{K-1}h[n]W_K^{in} \quad (20)$$

$$\hat{h}_P[n] = \frac{1}{K}\sum_{i=0}^{K-1}\hat{H}_P[i]W_K^{-in} = h[n] \quad (21)$$

As a result, time domain interpolation is a high-resolution method that is easy to realize. However, the number of subcarriers N must be the integral multiple of the pilot spacing m to use time domain interpolation, i.e., N/m must be an integer. m is determined by the maximum delay τ_m. The maximum pilot spacing that satisfies $\tau_m \leqslant K$ does not always satisfy the requirement of time domain interpolation. Smaller pilot spacing can be designed to satisfy both requirements, but it increases the number of pilot subcarriers and reduces system efficiency. To overcome the limitation of time domain interpolation, the modified time domain interpolation method is proposed.

2.2 Modified time domain interpolation

Considering the condition that N/m is not an integer, the first pilot tone is inserted at the first subcarrier, and the number of pilots $K = \langle N/m \rangle$. Assuming $\hat{H}_P[i] = H[im]$, then we have

$$\hat{H}_P[i] = H[im] = \sum_{n=0}^{N-1}h[n]W_N^{inm} \qquad i = 0, 1, \ldots, K-1 \quad (22)$$

Assuming $mK/N \approx 1$ and $W_N \approx W_{mK}$, similarly to Eqs. (20) and (21), we have

$$\hat{h}_P[n] = \frac{1}{K} \sum_{i=0}^{K-1} \hat{H}_P[i] W_K^{-in} \approx h[n] \qquad 0 \leqslant n \leqslant K-1 \tag{23}$$

The relationship between CIR $h[n]$ and the estimated CIR based on linear interpolation $\hat{h}_L[n]$ is derived from Eqs. (15) and (23) as follows:

$$h[n] \approx \frac{\hat{h}_L[n]}{P_{mK,m}[n]} \qquad n = 0, 1, \ldots, K-1 \tag{24}$$

$P_{mK,m}[n]$ is close to 0 when n is close to K, as shown in Fig. 1.

In Eq. (15), $I[n]$ in Eq. (14) is ignored. The error of Eq. (15) is given by

$$e = \left| \frac{\hat{h}_L[n]}{P_{mK,m}[n]} - \hat{h}_P[n] \right| = \frac{|I[n]|}{P_{mK,m}[n]} \tag{25}$$

The smaller the value of $P_{mK,m}[n]$, the greater the error caused by simplification in Eq. (15), and the error becomes significant when n is close to K. Thus, considering the accuracy of calculation and ease of implementation, the last quarter of $P_{mK,m}[n]$ with a smaller value is discarded, and the approximate calculation results of

$h[n]$ are as follows:

$$\hat{h}'_{\text{DFT}}[n] = \begin{cases} \dfrac{\hat{h}_L[n]}{P_{mK,m}[n]} & 0 \leqslant n \leqslant \langle \frac{3}{4}K \rangle - 1 \\[3mm] \dfrac{\hat{h}_L[n]}{P_{mK,m}[\langle 3/4K \rangle]} & \langle \frac{3}{4}K \rangle \leqslant n \leqslant K-1 \\[3mm] 0 & K \leqslant n \leqslant N-1 \end{cases} \tag{26}$$

where $\langle \frac{3}{4}K \rangle$ is the nearest integer with a value greater than $\frac{3}{4}K$. The estimated CFR at all subcarriers is obtained by the N-point FFT, given by

$$\hat{H}'_{\text{DFT}}[k] = \sum_{n=0}^{N-1} \hat{h}'_{\text{DFT}}[n] W_N^{nk} \qquad k = 0, 1, \ldots, N-1 \tag{27}$$

The calculation process of the proposed method is shown in Fig. 2.

3 Simulation Results and Discussion

3.1 Description of simulation

In this section, we analyze the performance of the proposed method on multipath channels. The OFDM system has 1 024 subcarriers and is modulated with 16-QAM. To
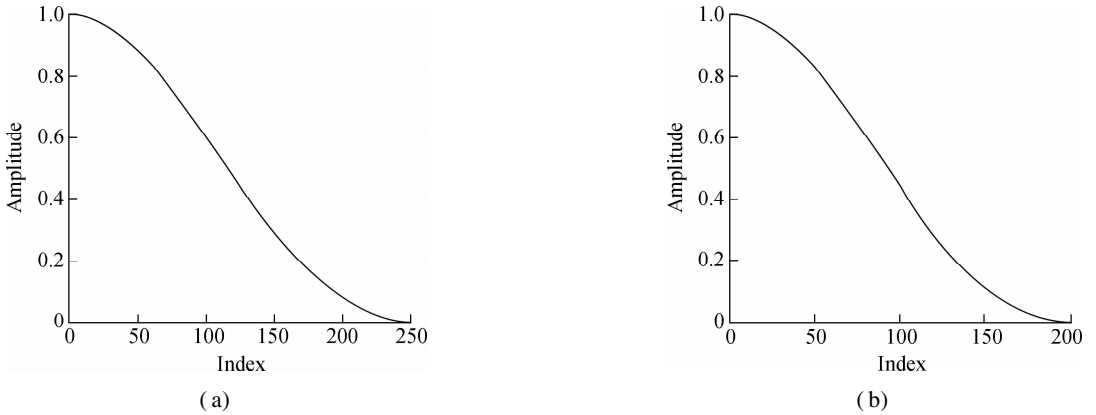
(a)   (b)

Fig. 1 The value of $P_{mK,m}[n]$ against index n. (a) $m=4$, $K=256$; (b) $m=5$, $K=205$
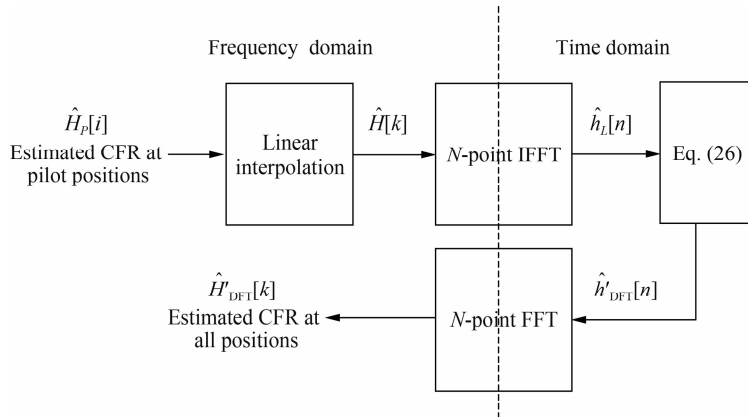
Fig. 2 Calculation process of the proposed method

investigate the performance of the proposed method separately, the synchronization is assumed to be perfect.

The channel is the Advanced Television Technology Center and the Grand Alliance DTV Laboratory's ensemble E model[16], and CIR for the static case is given by

$$h[n] = \delta[n] + 0.316\ 2\delta[n-2] + 0.199\ 5\delta[n-17] + \\ 0.129\ 6\delta[n-36] + 0.1\delta[n-75] + 0.1\delta[n-137]$$
$$(28)$$

The unit delay is assumed to be the same as the sample period; there are no power losses caused by the non-spaced sample[24].

For the channel with static CIR given in Eq. (28), the BER and mean square error (MSE) of $\{\hat{H}[k],\ k=0,\ 1,\ \ldots,\ 1\ 023\}$ at different signal noise ratios (SNR) are examined. The BER is obtained after the received data has been decoded as binary data. The initial data is binary data generated at random, and the number of simulations is 100. The LS estimations are compared based on comb-type pilots with different pilot spacings and interpolation methods.

3.2 Comparison with common interpolation methods

Figs. 3 and 4 show the BER and MSE performance of various interpolation methods against SNR. The legends "linear, cubic, spline, and proposed method" denote LS channel estimation based on comb-type pilots with linear interpolation, cubic interpolation, cubic spline interpolation, and modified time domain interpolation.
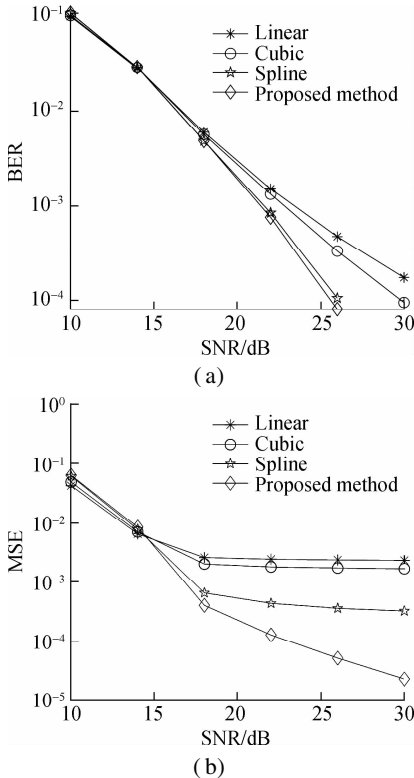

Fig.3 Comparing BER and MSE performance of linear, cubic, and cubic spline interpolation methods and the proposed method against SNR ($m=3$). (a) BER; (b) MSE
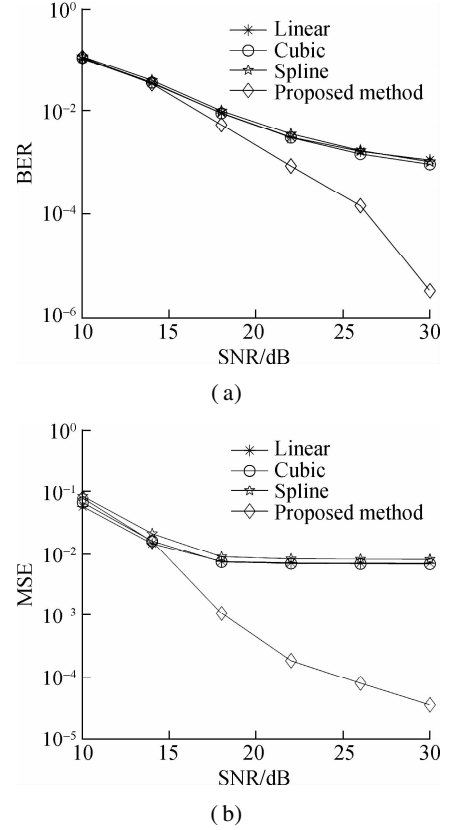

Fig.4 Comparing BER and MSE performance of linear, cubic, cubic spline interpolation methods and the proposed method against SNR ($m=4$). (a) BER; (b) MSE

It is obvious that

1) When the pilot spacing is small ($m=3$), the modified time domain interpolation outperforms linear interpolation and cubic interpolation in BER performance and has similar BER performance to cubic spline interpolation. Additionally, time domain interpolation is shown to be much better than other interpolation methods in MSE performance. The performance of linear interpolation is the worst.

2) When the pilot spacing is increased ($m=4$), the BER and MSE performance of cubic interpolation and cubic spline interpolation decrease rapidly, approaching linear interpolation, whereas the modified time domain interpolation method retains its good performance.

3.3 Comparison with time domain interpolation

When $m=4$, the time domain interpolation method meets the requirements for use. The legends "time domain interpolation, proposed method, and linear" denote LS channel estimation based on comb-type pilots with time domain interpolation, modified time domain interpolation, and linear interpolation in Fig. 5.

It is obvious that time domain interpolation and the proposed method provide comparable BER and MSE performance, which is significantly better than linear interpolation.
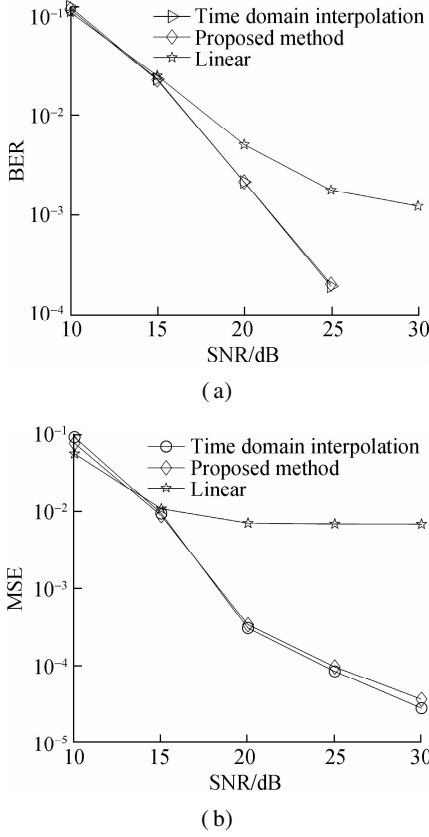
Fig. 5 Comparing BER and MSE performance of time domain interpolation, linear interpolation, and the proposed method against SNR ($m = 4$). (a) BER; (b) MSE

Time domain interpolation cannot be directly used when $m = 5$ and $m = 6$. The following methods are used to compare with the modified time domain interpolation.

Method 1 Perform K-point IDFT on the LS estimated CFR to obtain $\hat{h}_p[n]$ in the time domain, whose length is K, and extend it to an N-sample sequence by $(N - K)$-point zero-padding. Perform an N-point FFT on the obtained results to obtain the estimated CFR at all positions.

Method 2 Perform K-point IDFT on the LS estimated CFR to obtain $\hat{h}_p[n]$ in the time domain, whose length is K, and extend it to an mK-sample sequence by $(mK - K)$-point zero-padding. Perform mK-point DFT on the obtained result and take the previous N value as the estimated CFR of all positions.

The comparison between modified time domain interpolation, method 1, and method 2 is shown in Figs. 6 and 7. The legends "method 1, method 2, proposed method, linear" denote LS channel estimation based on comb-type pilots with method 1, method 2, the proposed method, and linear interpolation.

In the case of no noise, the CIR calculated by the modified time domain interpolation method (Eq. (26)) and the CIR calculated by performing K-point IDFT on the LS estimated CFR (Eq. (23)) are shown in Tabs. 1 and 2.
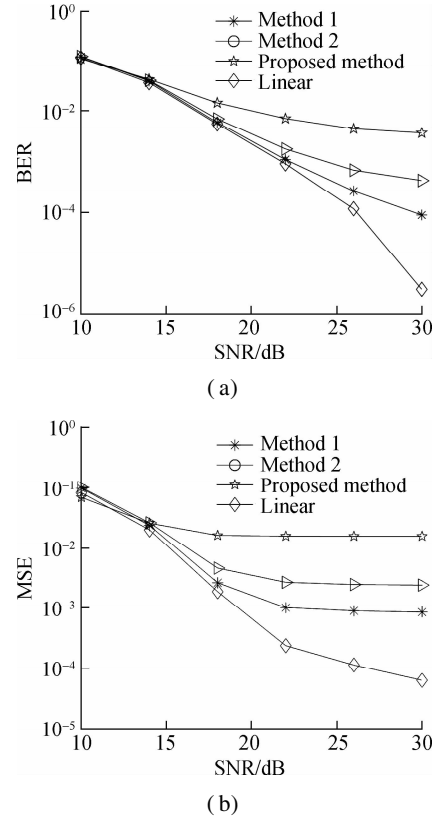
It can be seen that

Fig. 6 Comparing BER and MSE performance of linear interpolation, method 1, method 2, and the proposed method against SNR ($m = 5$). (a) BER; (b) MSE
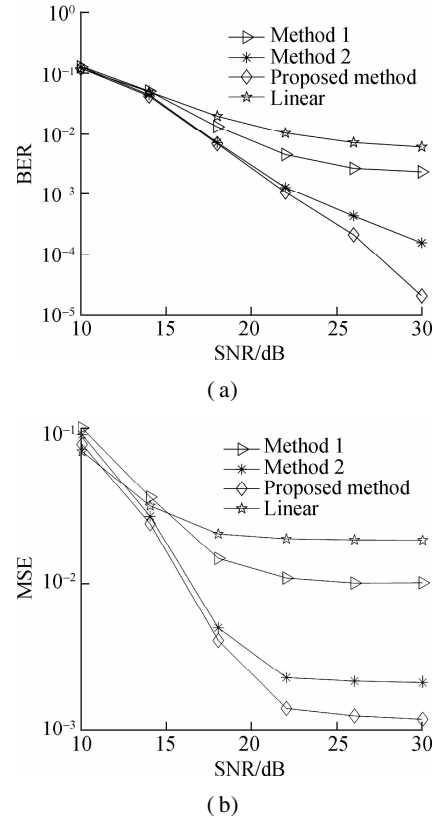

Fig. 7 Comparing BER and MSE performance of linear interpolation, method 1, method 2, and the proposed method against SNR ($m = 6$). (a) BER; (b) MSE

1) Methods 1 and 2 use K-point IDFT on the LS estimated CFR to calculate CIR; the amplitude of the estimated CIR is relatively accurate, but the phase error is significant and increases as index n increases.

2) Methods 1 and 2 outperform linear interpolation, but the mK-point DFT and K-point IDFT used do not meet the use conditions of the FFT algorithm (the number of FFT points must be a power of 2). Thus, the amount of calculation is very large. Methods 1 and 2 have low practical value and are only used as comparisons with the proposed method.

3) The modified time domain interpolation method outperforms methods 1 and 2 because it has similar calculation accuracy in the amplitude of the estimated CIR to methods 1 and 2, and the phase error is much smaller, as shown in Tabs. 1 and 2.

Tab. 1 CIR and the estimated CIR by the proposed method and IDFT ($m = 5$)

Index	CIR	Estimated CIR	
		Proposed method	IDFT
0	1	0.999 7	1.000 7
2	0.316 2	0.315 9∠0.002 0	0.316 6∠ −0.004 8
17	0.199 5	0.199 7∠0.003 3	0.199 7∠ −0.050 1
36	0.129 6	0.130 1∠0.005 5	0.129 4∠ −0.107 4
75	0.1	0.101 3∠0.009 2	0.099 1∠ −0.226 2
137	0.1	0.104 6∠0.020 2	0.097 0∠ −0.416 0

Tab. 2 CIR and the estimated CIR by the proposed method and IDFT ($m = 6$)

Index	CIR	Estimated CIR	
		Proposed method	IDFT
0	1	0.999 8∠0.001 3	1.001 3∠0.001 2
2	0.316 2	0.316 1∠0.003 9	0.317 0∠ −0.008 7
17	0.199 5	0.200 1∠0.005 4	0.199 8∠ −0.099 4
36	0.129 6	0.130 7∠0.007 9	0.128 7∠ −0.214 1
75	0.1	0.102 2∠0.012 4	0.096 5∠ −0.451 7
137	0.1	0.066 4∠0.044 7	0.088 4∠ −0.829 5

3.4 Computational complexity analysis

In practice, the reciprocals of $P_{mK,m}[n]$ can be stored, and division in Eq. (26) can be realized by multiplication. Using the proposed method, the computation amount includes three aspects: $(N-K)$-point linear interpolation, N-point IFFT, and Eq. (26). The computation amount for each step is shown in Tab. 3. $(2N - K + (N\log_2 N)/2)$ complex multiplications and $(N - K + N\log_2 N)$ complex additions are required. The proposed method has low computational complexity.

Tab. 3 Computation amount of the proposed method

Steps	Complex multiplication	Complex addition
Linear	$2(N-K)$	$N-K$
IFFT	$(N\log_2 N)/2$	$N\log_2 N$
Eq. (26)	K	0

4 Conclusions

1) LS channel estimation based on linear interpolation is theoretically deduced, and the relationship between the estimated CIR from LS based on linear interpolation and the ideal CIR is analyzed.

2) The modified time domain interpolation method is proposed, which uses CIR obtained by LS based on linear interpolation to calculate actual CIR, and interpolation is realized by zero-padding in the time domain, such as time domain interpolation.

3) LS estimations based on comb-type pilots of different pilot spacings with the proposed method, time domain interpolation, and interpolation methods in the frequency domain, such as linear interpolation, cubic interpolation, and cubic spline interpolation, are simulated and compared. The simulation results indicate that the proposed method performs similarly to and outperforms time domain interpolation with low complexity.

4) The proposed method is exempt from the requirement that the number of subcarriers is an integral multiple of pilot spacing. The consumption of subcarriers used for channel estimation can be reduced.

References

[1] Telagam N, Sahu P C, Panda S, et al. USRP based digital audio broadcasting using OFDM in virtual and remote laboratory[J]. *International Journal of Online and Biomedical Engineering* (*IJOE*), 2019, **15**(13): 77 − 85. DOI: 10. 3991/ijoe. v15i13. 8761.

[2] Jiang R, Wang X, Cao S, et al. Deep neural networks for channel estimation in underwater acoustic OFDM systems [J]. *IEEE Access*, 2019, **7**: 23579 − 23594. DOI: 10. 1109/ACCESS. 2019. 2899990.

[3] Honfoga A-C, Nguyen T T, Dossou M, et al. Application of FBMC to DVB-T2: A comparison vs classical OFDM transmissions[C]//*Proceedings of the* 2019 *IEEE Global Conference on Signal and Information Processing* (*GlobalSIP*). Ottawa, Canada, 2019: 1 − 5. DOI: 10. 1109/GlobalSIP45357. 2019. 8969550.

[4] Pandya S, Wakchaure M A, Shankar R, et al. Analysis of NOMA-OFDM 5G wireless system using deep neural network[J]. *The Journal of Defense Modeling and Simulation*: *Applications*, *Methodology*, *Technology*, 2021: 154851292199910. DOI: 10. 1177/1548512921999108.

[5] Sure P, Bhuma C M. A survey on OFDM channel estimation techniques based on denoising strategies[J]. *Engineering Science and Technology*, *an International Journal*, 2017, **20**(2): 629 − 636. DOI: 10. 1016/j. jestch. 2016. 09. 011.

[6] Sutar M B, Patil V S. LS and MMSE estimation with different fading channels for OFDM system[C]//*Proceedings of the* 2017 *International Conference of Electronics, Communication and Aerospace Technology* (*ICECA*). Coimbatore, India, 2017: 740 − 745. DOI: 10. 1109/ICECA. 2017. 8203641.

[7] Coleri S, Ergen M, Puri A, et al. Channel estimation techniques based on pilot arrangement in OFDM systems [J]. *IEEE Transactions on Broadcasting*, 2002, **48**(3):

223 – 229. DOI: 10. 1109/TBC. 2002. 804034.

[8] Jia L, Cao D Z, Hou C P. Two novel transform domain estimation methods for OFDM system and their application environment[C]//*Canadian Conference on Electrical and Computer Engineering* 2004. Niagara Falls, ON, Canada, 2004: 377 – 380. DOI: 10. 1109/CCECE. 2004. 1345034.

[9] Rocha C A, Mendes L L, Da Silveira A C. Performance analysis of channel estimation schemes for OFDM systems [C]//*Proceedings of International Workshop on Telecommunications—IWT*-2007. Santa Rita do Sapucaí, Brasil, 2007: 32 – 36.

[10] Li D, Guo F, Li G S, et al. Enhanced DFT interpolation-based channel estimation for OFDM systems with virtual subcarriers[C]//*2006 IEEE 63rd Vehicular Technology Conference*. Melbourne, Vic, Australia, 2006: 1580 – 1584. DOI: 10. 1109/VETECS. 2006. 1683112.

[11] Zheng K, Su J, Wang W B. DFT-based channel estimation in comb-type pilot-aided OFDM systems with virtual carriers[C]//*2007 IEEE 18th International Symposium on Personal, Indoor and Mobile Radio Communications*. Athens, Greece, 2007: 1 – 5. DOI: 10. 1109/PIMRC. 2007. 4394389.

[12] Zhang L, Hong Z H, Thibault L. Improved DFT-based channel estimation for OFDM systems with null subcarriers[C]//*2009 IEEE 70th Vehicular Technology Conference Fall*. Anchorage, AK, USA, 2009: 1 – 5. DOI: 10. 1109/VETECF. 2009. 5378722.

[13] Jawhar Y A, Audah L, Taher M A, et al. A review of partial transmit sequence for PAPR reduction in the OFDM systems[J]. *IEEE Access*, 2019, **7**: 18021 – 18041. DOI: 10. 1109/ACCESS. 2019. 2894527.

[14] Edfors O, Sandell M, van de Beek J-J, et al. Analysis of DFT-based channel estimators for OFDM [J]. *Wireless Personal Communications*, 2000, **12**(1): 55 – 70. DOI: 10. 1109/26. 966074.

[15] Kang Y, Kim K, Park H. Efficient DFT-based channel estimation for OFDM systems on multipath channels[J]. *IET Communications*, 2007, **1**(2): 197 – 202. DOI: 10. 1049/iet-com: 20050337.

[16] Minn H, Bhargava V K. An investigation into time-do-main approach for OFDM channel estimation[J]. *IEEE Transactions on Broadcasting*, 2000, **46**(4): 240 – 248. DOI: 10. 1109/11. 898744.

[17] Murad M, Tasadduq I A, Otero P. Towards multicarrier waveforms beyond OFDM: Performance analysis of GFDM modulation for underwater acoustic channels[J]. *IEEE Access*, 2020, **8**: 222782 – 222799. DOI: 10. 1109/ ACCESS. 2020. 3043718.

[18] Luo J F, Xie Z J, Xie M. Interpolated DFT algorithms with zero padding for classic windows[J]. *Mechanical Systems and Signal Processing*, 2016, **70/71**: 1011 – 1025. DOI: 10. 1016/j. ymssp. 2015. 09. 045.

[19] Simeone O, Spagnolini U. Adaptive pilot pattern for OFDM systems[C]//*2004 IEEE International Conference on Communications*. Paris, France, 2004: 978 – 982. DOI: 10. 1109/ICC. 2004. 1312647.

[20] Jiang R K, Wang X T, Cao S, et al. Joint compressed sensing and enhanced whale optimization algorithm for pilot allocation in underwater acoustic OFDM systems[J]. *IEEE Access*, 2019, **7**: 95779 – 95796. DOI: 10. 1109/ ACCESS. 2019. 2929305.

[21] Qiao G, Babar Z, Ma L, et al. MIMO-OFDM underwater acoustic communication systems—A review[J]. *Physical Communication*, 2017, **23**: 56 – 64. DOI: 10. 1016/j. phycom. 2017. 02. 007.

[22] Barnett A H, Magland J, af Klinteberg L. A parallel non-uniform fast Fourier transform library based on an "exponential of semicircle"kernel[J]. *SIAM Journal on Scientific Computing*, 2019, **41**(5): C479 – C504. DOI: 10. 1137/ 18m120885x.

[23] Zhao Y P, Huang A P. A novel channel estimation method for OFDM mobile communication systems based on pilot signals and transform-domain processing [C]//*1997 IEEE 47th Vehicular Technology Conference on Technology in Motion*. Phoenix, AZ, USA, 1997: 2089 – 2093. DOI: 10. 1109/VETEC. 1997. 605966.

[24] Fan T L, Wen Y C, Huang S B, et al. An improved DFT-based channel estimation algorithm for OFDM system in non-sample-spaced multipath channels[J]. *Circuits, Systems, and Signal Processing*, 2014, **33**(4): 1277 – 1288. DOI: 10. 1007/s00034-013-9678-0.

OFDM 通信系统 LS 信道估计的改进时域插值法

张烨楠　　许飞云　　贾民平

（东南大学机械工程学院，南京 211189）

摘要：对于正交频分复用（OFDM）系统，针对基于梳状导频的最小二乘信道估计算法中时域插值法无法灵活选择导频间隔的问题，提出了一种改进的时域插值法. 首先，使用梳状导频和最小二乘法信道估计算法得到频域的导频处信道估计值，通过线性插值和反快速傅里叶变换得到时域的信道冲击响应估计值；其次，通过理论推导分析线性插值得到的信道冲击响应估计值的误差，并提出对其进行修正的方法；最后，通过对修正后的信道冲击响应进行时域补零和快速傅里叶变换，得到频域所有子载波处的信道估计值. 仿真结果表明，所提方法与时域插值法性能相似，且不必满足时域插值法子载波数为导频间隔整数倍的要求. 所提方法能灵活选择导频间隔，减少导频数目和信道估计对子载波资源的消耗.

关键词：正交频分复用（OFDM）；信道估计；线性插值；时域插值

中图分类号：TN929. 5