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Disturbance rejection control for uncertain systems with actuator saturation

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Abstract: To address the issue that conventional disturbance observer (DO) design did not consider actuator saturation, which is prevalent in practical systems, a DO-based control (DOBC) strategy with anti-windup compensation is proposed. First, the reason of windup phenomenon in the conventional DOBC when the actuator is saturated is studied. Then, an anti-windup compensator is designed by minimizing the performance index, and patched to the DO so that the modified DOBC can effectively handle actuator saturation. Finally, local asymptotic stability analysis is performed on the resulting closed-loop systems. Comparative simulation results show that when there is actuator saturation, the proposed method has smaller errors in position tracking and disturbance estimation, and the designed compensator can maintain the DO states to be as close as possible to those without actuator saturation. This verifies that the proposed method is superior in anti-disturbance and anti-windup.

Key words: anti-windup; disturbance observer; dynamic compensator; actuator saturation

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In practical control systems, external disturbances and internal parameter uncertainties are inevitable, leading to adverse effects on control performance or system stability[1−3]. Attenuating their adverse effect through feedback control has received considerable attention across many applications, from motion control[4] to attitude control[5]. However, the actuator has limited operating range to implement the calculated control law[6]. This makes disturbance rejection control more challenging because actuator saturation alone may lead to performance degradation, such as large overshoot, limit circle, or even divergence. Therefore, addressing the challenge of rejecting disturbances and uncertainties using saturated control is imperative for theoretical developments and practical applications.

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To attenuate the adverse effects of disturbances and uncertainties, several approaches have been developed, which can be categorized into passive and active approaches. Robust control and sliding mode control are typical passive approaches where robustness against uncertainties is considered at the outset of the control design. Disturbance observer-based control (DOBC)[1] and active disturbance rejection control (ADRC)[4] are typical active approaches, where a disturbance estimator rejecting disturbances and suppressing uncertainties is designed and augmented with the nominal control addressing nominal performance specifications and stability. The latter approach has received considerable attention owing to its promising features, such as the preservation of the nominal performance and the “separation principle”[2].

There are two main approaches for handling actuator saturation[7]. The first approach considers control constraints at the outset of control design (e. g., nested feedback design[8] and model predictive control[9]). The other approach designs a nominal controller for handling nominal control performance/specifications without actuator saturation and designs an anti-windup compensator that attenuates the effect of actuator saturation[10−11]. The latter approach has received considerable attention owing to its fine properties, such as nominal control performance recovery. Recent work on the latter approach is primarily focused on stability analysis under an anti-windup controller, i. e., designing an anti-windup gain that maximizes an estimation for the domain of attraction (or domain of performance with level γ) of the closed-loop system[7,11].

However, there is limited work on disturbance rejection control using saturated controllers, although this problem is receiving increasing attention[12−16]. Nguyen et al.[12] considered disturbance attenuation with input saturation by incorporating the saturation nonlinearity into the control design using linear matrix inequality. However, the disturbances therein are assumed to have a limited H_∞ norm. Errouissi et al.[13] and An et al.[14] conducted different modifications on disturbance observers for practical applications. However, these approaches may lack theoretical analysis, such as the effect of disturbances on the stability region. Ran et al.[15] mainly considered how the conventional ADRC can guarantee the local stabilization with actuator saturation by replacing $\boldsymbol{u}$ with $\mathrm{sat}(\boldsymbol{u})$ and how to estimate the region of attraction for linear systems. Yu et al.[16] investigated disturbance rejection control for the networked control systems with actuator saturation to achieve input-to-state stability. It is worth noting that the result[15-16] is a type of static anti-windup controller.

In this study, we consider disturbance rejection control for uncertain systems with actuator saturation. A conventional DOBC is introduced to preserve the nominal performance of uncertain systems. Subsequently, a dynamic anti-windup compensator is designed and patched into DOBC for actuator saturation mitigation. The dynamic compensator is derived in a natural way using an optimal approach with a reasonable performance index[17], which can maintain the DO states as close as possible to those without input saturation. Inspired by the results[18], local asymptotical stability analysis is performed to evaluate the effects of unknown disturbances on the stability region.

1 Disturbance Observer-Based Control

Here, consider an uncertain system with actuator saturation as follows:

$$\left.\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\,\mathrm{sat}(\boldsymbol{u}) + \boldsymbol{B}_2\boldsymbol{d}\\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x}\end{aligned}\right\} \tag{1}$$

where $\boldsymbol{x}\in\mathbf{R}^n$, $\boldsymbol{y}\in\mathbf{R}^m$, $\boldsymbol{u}\in\mathbf{R}^q$, and $\boldsymbol{d}\in\mathbf{R}^p$ are the system state, controlled output, control input, and unknown disturbances/uncertainties, respectively; $\boldsymbol{A}$, $\boldsymbol{B}_1$, $\boldsymbol{B}_2$, $\boldsymbol{C}$ are the system, input, disturbance, and measurement matrices with admissible dimensions, respectively. The actuator $\mathrm{sat}(\boldsymbol{u})$ is modeled as follows:

$$\mathrm{sat}(u_i) = \begin{cases} u_i & |u_i| \leqslant u_{\lim_i} \\ \mathrm{sign}(u_i)\,u_{\lim_i} & \text{otherwise}\end{cases}$$

where $u_{\lim_i}$ is the maximum amplitude of the i-th actuator, and $\mathrm{sign}(u_i)$ is the sign of the i-th control input. The objective is to regulate output $\boldsymbol{y}$ to a reference $\boldsymbol{r}$ while rejecting $\boldsymbol{d}$ using saturated control input $\mathrm{sat}(\boldsymbol{u})$.

1.1 DOBC without saturation

To reject disturbances $\boldsymbol{d}$ on controlled output $\boldsymbol{y}$ without input saturation (i.e., $\mathrm{sat}(\boldsymbol{u})\to\boldsymbol{u}$), a DO[1] is designed to estimate $\boldsymbol{d}$, which takes the following form:

$$\left.\begin{aligned}\dot{\bar{z}} &= -\boldsymbol{L}\boldsymbol{B}_2(\bar{z} + \boldsymbol{L}\bar{\boldsymbol{x}}) - \boldsymbol{L}(\boldsymbol{A}\bar{\boldsymbol{x}} + \boldsymbol{B}_1\boldsymbol{u})\\ \hat{\boldsymbol{d}} &= \bar{z} + \boldsymbol{L}\bar{\boldsymbol{x}}\end{aligned}\right\} \tag{2}$$

where $\bar{\boldsymbol{x}}\in\mathbf{R}^n$ and $\bar{z}\in\mathbf{R}^p$ are the system state and internal state of DO without actuator saturation; $\boldsymbol{L}$ is the gain matrix; $\hat{\boldsymbol{d}}$ is the estimated disturbance. Moreover, one can obtain the dynamic of disturbance estimation as follows:

$$\dot{\hat{\boldsymbol{d}}} = -\boldsymbol{L}\boldsymbol{B}_2\hat{\boldsymbol{d}} + \boldsymbol{L}\boldsymbol{B}_2\boldsymbol{d} \tag{3}$$

By defining disturbance estimation error as $\boldsymbol{e}_d = \boldsymbol{d} - \hat{\boldsymbol{d}}$ and using Eqs. (1)-(3), we obtain

$$\dot{\boldsymbol{e}}_d = -\boldsymbol{L}\boldsymbol{B}_2\boldsymbol{e}_d + \dot{\boldsymbol{d}} \tag{4}$$

where $-\boldsymbol{L}\boldsymbol{B}_2$ satisfies the following assumption.

Assumption 1 $\boldsymbol{L}$ is designed such that all the eigenvalues of $-\boldsymbol{L}\boldsymbol{B}_2$ have negative real parts.

This assumption can be satisfied as long as $\boldsymbol{B}_2$ is of full column rank; otherwise, the redundant variables in $\boldsymbol{d}$ can be removed such that $\boldsymbol{B}_2$ is of full column rank[19].

A composite DOBC law can be designed as follows:

$$\boldsymbol{u} = \boldsymbol{K}_x\bar{\boldsymbol{x}} + \boldsymbol{K}_d\hat{\boldsymbol{d}} \tag{5}$$

where gain matrices $\boldsymbol{K}_x, \boldsymbol{K}_d$ are determined for state regulation and disturbance rejection, respectively.

Substituting Eq. (5) into Eq. (1) gives

$$\begin{aligned}\dot{\bar{\boldsymbol{x}}} &= \boldsymbol{A}\bar{\boldsymbol{x}} + \boldsymbol{B}_1\boldsymbol{K}_x\bar{\boldsymbol{x}} + \boldsymbol{B}_1\boldsymbol{K}_d\hat{\boldsymbol{d}} + \boldsymbol{B}_2\boldsymbol{d} = \\ &(\boldsymbol{A} + \boldsymbol{B}_1\boldsymbol{K}_x)\bar{\boldsymbol{x}} + \boldsymbol{B}_1\boldsymbol{K}_d(\boldsymbol{d} - \boldsymbol{e}_d) + \boldsymbol{B}_2\boldsymbol{d} = \\ &(\boldsymbol{A} + \boldsymbol{B}_1\boldsymbol{K}_x)\bar{\boldsymbol{x}} - \boldsymbol{B}_1\boldsymbol{K}_d\boldsymbol{e}_d + (\boldsymbol{B}_1\boldsymbol{K}_d + \boldsymbol{B}_2)\boldsymbol{d} = \\ &(\boldsymbol{A} + \boldsymbol{B}_1\boldsymbol{K}_x)\bar{\boldsymbol{x}} + \boldsymbol{B}_d\boldsymbol{d} - \boldsymbol{B}_1\boldsymbol{K}_d\boldsymbol{e}_d\end{aligned} \tag{6}$$

where $\boldsymbol{B}_d = \boldsymbol{B}_1\boldsymbol{K}_d + \boldsymbol{B}_2$.

Using Eq. (4) and the stability of $-\boldsymbol{L}\boldsymbol{B}_2$, $\dot{\boldsymbol{d}}\to\boldsymbol{O}$ implies that $\boldsymbol{e}_d$ converges to $\boldsymbol{O}$ regardless of system state $\boldsymbol{x}$. After a short convergence time of linear DO (Eq. (2)), where the convergence rate can be tuned by appropriately designing the DO gain matrix $\boldsymbol{L}$, Eq. (6) reduces to

$$\dot{\bar{\boldsymbol{x}}} = (\boldsymbol{A} + \boldsymbol{B}_1\boldsymbol{K}_x)\bar{\boldsymbol{x}} + \boldsymbol{B}_d\boldsymbol{d} \tag{7}$$

To guarantee the stability and disturbance rejection performance in the output channel, gain matrices $\boldsymbol{K}_x$ and $\boldsymbol{K}_d$ are designed to satisfy the following assumptions simultaneously[1]:

Assumption 2 $\boldsymbol{A} + \boldsymbol{B}_1\boldsymbol{K}_x$ is Hurwitz to guarantee stability.

Assumption 3 $\boldsymbol{C}[-(\boldsymbol{A} + \boldsymbol{B}_1\boldsymbol{K}_x)]^{-1}\boldsymbol{B}_d = \boldsymbol{O}$ holds to satisfy the disturbance rejection condition.

Assumption 2 guarantees system stability without disturbances, which can be satisfied by the controllability of the matrix pair $(\boldsymbol{A}, \boldsymbol{B}_1)$, and Assumption 3 handles disturbance rejection performance. The left-hand side of the equation in Assumption 3 is obtained through the transfer function (with $s = 0$) from disturbances $\boldsymbol{d}$ to output $\boldsymbol{y}$ of Eq. (7). Therefore, DOBC (Eq. (5)) with DO (Eq. (2)) can reject the effect of disturbances on output in the steady state.

Combining Eqs. (4) and (6), one can obtain the composite state and disturbance estimation error as follows:

$$\begin{bmatrix}\dot{\boldsymbol{e}}_d\\ \dot{\bar{\boldsymbol{x}}}\end{bmatrix} = \begin{bmatrix}-\boldsymbol{L}\boldsymbol{B}_2 & \boldsymbol{O}\\ -\boldsymbol{B}_1\boldsymbol{K}_d & \boldsymbol{A} + \boldsymbol{B}_1\boldsymbol{K}_x\end{bmatrix}\begin{bmatrix}\boldsymbol{e}_d\\ \bar{\boldsymbol{x}}\end{bmatrix} + \begin{bmatrix}\boldsymbol{I} & \boldsymbol{O}\\ \boldsymbol{O} & \boldsymbol{B}_d\end{bmatrix}\begin{bmatrix}\dot{\boldsymbol{d}}\\ \boldsymbol{d}\end{bmatrix}$$

where $\boldsymbol{A}_e = \begin{bmatrix}-\boldsymbol{L}\boldsymbol{B}_2 & \boldsymbol{O}\\ -\boldsymbol{B}_1\boldsymbol{K}_d & \boldsymbol{A} + \boldsymbol{B}_1\boldsymbol{K}_x\end{bmatrix}$.

Considering $\boldsymbol{A}_e$ is Hurwitz, one can prove its stability using the input-to-state stability theory.

1.2 Windup phenomenon

Under input saturation, disturbance estimation and its error using DO (Eq. (2)) are given by

$$\left.\begin{aligned}\dot{\hat{d}} &= -LB_2\hat{d} + LB_2 d + LB_1[\mathrm{sat}(u) - u]\\ \dot{e}_d &= -LB_2 e_d - LB_1 D_u + \dot{d}\end{aligned}\right\}\qquad(8)$$

where $D_u = \mathrm{sat}(u) - u$.

Both are subjected to the effect of the differences between saturated input and calculated input D_u. The resulting closed-loop system with actuator saturation using DOBC control law (Eq. (5)) becomes

$$\begin{aligned}\dot{x} &= Ax + B_1\mathrm{sat}(u) + B_2 d =\\ &Ax + B_1 u + B_2 d + B_1[\mathrm{sat}(u) - u] =\\ &(A + B_1K_x)x + B_d d + B_1 D_u - B_1K_d e_d\end{aligned}\qquad(9)$$

where $\eta_{\mathrm{effect}} = B_1D_u - B_1K_de_d$.

Because e_d in Eq. (8) is a function of D_u, $-B_1K_de_d$ denotes the effect of disturbance observer (without considering input saturation) on the closed-loop system. Additionally, the term η_{effect} in Eq. (9) denotes the lumped effect of input saturation on closed-loop systems.

Suppose $B_1 = B_2$, i.e., the disturbances satisfying the so-called matching condition, then $K_d = -I$. We first consider that the control input saturates at positive upper bound, i.e., $D_u = \mathrm{sat}(u) - u \leqslant O$, which is either due to a reference with a positive value or disturbances with a negative value. For both cases, the amplitude of disturbance estimation $\hat{d}$ will be larger than the real disturbance. Consequently, the calculated control amplitude in Eq. (5) becomes larger. This results in windup due to actuator saturation.

2 Windup Augmentation

In this section, the main results are given with the overall diagram in Fig. 1. The block within the dotted square denotes the modified DO.

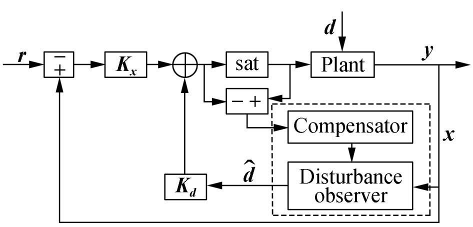

Fig. 1 DOBC augmented with anti-windup

2.1 Closed-loop dynamics without saturation

We first derive a compact form of the closed-loop system (Eq. (1)) without actuator saturation under DOBC (Eq. (5)) with DO (Eq. (2)). The DO dynamic (Eq. (2)) can be written in the following form:

$$\begin{aligned}\dot{\bar{z}} &= -LB_2(\bar{z} + L\bar{x}) - L[A\bar{x} + B_1(K_x\bar{x} + K_d(\bar{z} + L\bar{x}))] =\\ &(-LB_2 - LB_1K_d)\bar{z} + (-LB_2L - LA - LB_1K_x - LB_1K_dL)\bar{x}\end{aligned}$$

where $A_z = -LB_2 - LB_1K_d$, $B_z = -LB_2L - LA - LB_1K_x - LB_1K_dL$.

Consequently, the DOBC control law without actuator saturation is given by

$$\left.\begin{aligned}\dot{\bar{z}} &= A_z\bar{z} + B_z\bar{x}\\ u &= K_d\bar{z} + (K_x + K_dL)\bar{x}\end{aligned}\right\}\qquad(10)$$

which can be seen as a dynamic controller. Under this control law, the closed-loop system without input saturation is given by

$$\begin{aligned}\dot{\bar{x}} &= A\bar{x} + B_1u + B_2d =\\ &A\bar{x} + B_1(K_d\bar{z} + K_x\bar{x} + K_dL\bar{x}) + B_2d =\\ &(A + B_1K_x + B_1K_dL)\bar{x} + B_1K_d\bar{z} + B_2d\end{aligned}$$

where $A_c = A + B_1K_x + B_1K_dL$, $B_c = B_1K_d$.

The closed-loop system, including state dynamics and controller without input saturation, is given by

$$\begin{bmatrix}\dot{\bar{z}}\\ \dot{\bar{x}}\end{bmatrix} = \begin{bmatrix}A_z & B_z\\ B_c & A_c\end{bmatrix}\begin{bmatrix}\bar{z}\\ \bar{x}\end{bmatrix} + \begin{bmatrix}O\\ B_2d\end{bmatrix}\qquad(11)$$

where $\dot{\bar{Y}} = \begin{bmatrix}\dot{\bar{z}}\\ \dot{\bar{x}}\end{bmatrix}$, $A_Y = \begin{bmatrix}A_z & B_z\\ B_c & A_c\end{bmatrix}$.

The asymptotical stability of matrix A_Y is derived as follows. By performing a similarity transformation on matrix A_Y, we obtain

$$\begin{bmatrix}I & L\\ O & I\end{bmatrix}A_Y\begin{bmatrix}I & -L\\ O & I\end{bmatrix} = \begin{bmatrix}-LB_2 & O\\ B_1K_d & A + B_1K_x\end{bmatrix}$$

which means that the eigenvalues of A_Y are determined by the eigenvalues of $-LB_2$ and $A + B_1K_x$. Consequently, if the eigenvalues of $-LB_2$ and $A + B_1K_x$ are negative, matrix A_Y is Hurwitz. Actually, the matrix A_Y being Hurwitz should always be guaranteed to ensure the stability of the unsaturated control system by following Assumptions 1 and 2.

2.2 Closed-loop dynamics with saturation

With actuator saturation, the constrained x-dynamics corresponding to Eq. (11) is given by

$$\begin{aligned}\dot{x} &= Ax + B_1\mathrm{sat}(u) + B_2d =\\ &Ax + B_1u + B_1D_u + B_2d =\\ &A_cx + B_cz + B_2d + B_1D_u\end{aligned}\qquad(12)$$

As shown in Eq. (12), once the controller structure (Eq. (5)) with gain matrices K_x, K_d is predesigned, there is no design freedom in A_c and B_c. Additionally, B_2d and B_1D_u cannot be changed. Therefore, the only possibility is to redesign z in Eq. (12). Note that by changing z, the disturbance estimation, $\hat{d}$, will change and u and D_u will also change.

Without loss of generality, following the common technique in anti-windup control[17], an extra term, $\boldsymbol{\xi}$, is added to the conventional DOBC (Eq. (10)) to attenuate the effect of actuator saturation. The modified DOBC is given as follows:

$$\left.\begin{aligned} \dot{z} &= \boldsymbol{A}_z z + \boldsymbol{B}_z \boldsymbol{x} - \boldsymbol{\xi} \\ \boldsymbol{u} &= \boldsymbol{K}_d z + (\boldsymbol{K}_x + \boldsymbol{K}_d \boldsymbol{L})\boldsymbol{x} \end{aligned}\right\} \tag{13}$$

where $\boldsymbol{\xi}$ will be further determined.

Based on the modified DOBC (Eq. (13)), the closed-loop system with input saturation is governed by

$$\begin{bmatrix} \dot{z} \\ \dot{\boldsymbol{x}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_z & \boldsymbol{B}_z \\ \boldsymbol{B}_c & \boldsymbol{A}_c \end{bmatrix} \begin{bmatrix} z \\ \boldsymbol{x} \end{bmatrix} + \begin{bmatrix} -\boldsymbol{\xi} \\ \boldsymbol{B}_2 \boldsymbol{d} + \boldsymbol{B}_1 \boldsymbol{D}_u \end{bmatrix} \tag{14}$$

where $\dot{\boldsymbol{Y}} = \begin{bmatrix} \dot{z} \\ \dot{\boldsymbol{x}} \end{bmatrix}$.

2.3 Dynamic compensator

As discussed above, there are generally two approaches for handling actuator saturation[7]. Rather than dealing with actuator saturation at the outset of control design[12], the latter two-step-based approach is adopted. It is assumed that the pre-designed DOBC (Eq. (10)) can guarantee satisfying closed-loop performance without actuator saturation. $\boldsymbol{\xi}$ is also designed so that the closed-loop dynamic (Eq. (14)) with actuator saturation gets as close as possible to the closed-loop dynamic (Eq. (11)) without actuator saturation.

By defining $\boldsymbol{e}_z = z - \bar{z}$ and $\boldsymbol{e}_x = \boldsymbol{x} - \bar{\boldsymbol{x}}$ and using Eqs. (11) and (14), error dynamics $\boldsymbol{e}_Y = \begin{bmatrix} \boldsymbol{e}_z \\ \boldsymbol{e}_x \end{bmatrix}$ are governed by

$$\begin{bmatrix} \dot{\boldsymbol{e}}_z \\ \dot{\boldsymbol{e}}_x \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_z & \boldsymbol{B}_z \\ \boldsymbol{B}_c & \boldsymbol{A}_c \end{bmatrix} \begin{bmatrix} \boldsymbol{e}_z \\ \boldsymbol{e}_x \end{bmatrix} + \begin{bmatrix} -\boldsymbol{\xi} \\ \boldsymbol{B}_1 \boldsymbol{D}_u \end{bmatrix} \tag{15}$$

To investigate the effect of $\boldsymbol{\xi}$ and $\boldsymbol{B}_1\boldsymbol{D}_u$ on $\boldsymbol{e}_z$ and $\boldsymbol{e}_x$, we perform Laplace transformation on Eq. (15) as follows:

$$\boldsymbol{e}_Y(s) = (s\boldsymbol{I} - \boldsymbol{A}_Y)^{-1}\boldsymbol{e}_Y(0) + (s\boldsymbol{I} - \boldsymbol{A}_Y)^{-1}\begin{bmatrix} -\boldsymbol{\xi} \\ \boldsymbol{B}_1\boldsymbol{D}_u \end{bmatrix} \tag{16}$$

Because $\boldsymbol{e}_Y(0)$ has no effect on the final result due to $\boldsymbol{A}_Y$ being Hurwitz and it can always be chosen such that $\boldsymbol{e}_Y(0) = \boldsymbol{O}$ with $z(0) = \bar{z}(0)$ and $\boldsymbol{x}(0) = \bar{\boldsymbol{x}}(0)$, it is omitted in the following derivation.

It is highly desirable to attenuate or remove the effect of $\boldsymbol{B}_1\boldsymbol{D}_u$ on $\boldsymbol{e}_x$ by appropriately choosing $\boldsymbol{\xi}$. To this end, we define

$$(s\boldsymbol{I} - \boldsymbol{A}_Y)^{-1} = \begin{bmatrix} s\boldsymbol{I} - \boldsymbol{A}_z & -\boldsymbol{B}_z \\ -\boldsymbol{B}_c & s\boldsymbol{I} - \boldsymbol{A}_c \end{bmatrix}^{-1} = \begin{bmatrix} \boldsymbol{\Omega}_{11} & \boldsymbol{\Omega}_{12} \\ \boldsymbol{\Omega}_{21} & \boldsymbol{\Omega}_{22} \end{bmatrix}$$

where

$$\boldsymbol{\Omega}_{11} = (s\boldsymbol{I} - \boldsymbol{A}_z - \boldsymbol{B}_z(s\boldsymbol{I} - \boldsymbol{A}_c)^{-1}\boldsymbol{B}_c)^{-1},\ \boldsymbol{\Omega}_{12} = \boldsymbol{\Omega}_{11}\boldsymbol{B}_z(s\boldsymbol{I} - \boldsymbol{A}_c)^{-1}$$
$$\boldsymbol{\Omega}_{21} = \boldsymbol{\Omega}_{22}\boldsymbol{B}_c(s\boldsymbol{I} - \boldsymbol{A}_z)^{-1},\ \boldsymbol{\Omega}_{22} = (s\boldsymbol{I} - \boldsymbol{A}_c - \boldsymbol{B}_c(s\boldsymbol{I} - \boldsymbol{A}_z)^{-1}\boldsymbol{B}_z)^{-1}$$

Then, we can obtain the transfer function from $\begin{bmatrix} -\boldsymbol{\xi} \\ \boldsymbol{B}_1\boldsymbol{D}_u \end{bmatrix}$ to $\boldsymbol{e}_x(s)$ using Eq. (16). To ensure that the transfer function is zero, the following condition must be satisfied:

$$[\boldsymbol{O} \quad \boldsymbol{I}](s\boldsymbol{I} - \boldsymbol{A}_Y)^{-1}\begin{bmatrix} \boldsymbol{\xi} \\ \boldsymbol{O} \end{bmatrix} = [\boldsymbol{O} \quad \boldsymbol{I}](s\boldsymbol{I} - \boldsymbol{A}_Y)^{-1}\begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{B}_1\boldsymbol{D}_u \end{bmatrix}$$

which is equivalent to

$$\boldsymbol{\Omega}_{21}\boldsymbol{\xi} = \boldsymbol{\Omega}_{22}\boldsymbol{B}_1\boldsymbol{D}_u \tag{17}$$

Using the relation between $\boldsymbol{\Omega}_{21}$ and $\boldsymbol{\Omega}_{22}$, Eq. (17) is equivalent to

$$\boldsymbol{B}_c(s\boldsymbol{I} - \boldsymbol{A}_z)^{-1}\boldsymbol{\xi} = \boldsymbol{B}_1\boldsymbol{D}_u$$

However, matrix $\boldsymbol{B}_c = \boldsymbol{B}_1\boldsymbol{K}_d$ is rank-deficient, except for the trivial scalar case; therefore, $\boldsymbol{\xi}$ cannot be determined uniquely. Even $\det(\boldsymbol{B}_c) \neq 0$, how to obtain $\boldsymbol{\xi}$ in state space is still an open problem.

An alternative approach is exploited to generate a simple but reasonable $\boldsymbol{\xi}$. Park et al.[20] showed that the main reason for performance degradation under input saturation is because of the difference in the controller state between the saturated and unsaturated systems. Thus, in anti-windup design, it is reasonable to derive $\boldsymbol{\xi}$ in Eq. (13) by minimizing the effect of the above differences, i. e., minimizing $\boldsymbol{e}_z$ rather $\boldsymbol{e}_x$. Inspired by the optimal approach[17], we define an optimal index to derive the explicit form of $\boldsymbol{\xi}$. The optimal index is chosen as the function of $\boldsymbol{e}_z$, which is given by

$$J = \int_0^{\infty} \| \boldsymbol{e}_z \|^2 \mathrm{d}t \tag{18}$$

Using Parseval's theorem, the performance index in Eq. (18) is equivalent to

$$J = \frac{1}{2\pi \mathrm{j}}\int_{-\mathrm{j}\infty}^{\mathrm{j}\infty} \| \boldsymbol{e}_z(s) \|^2 \mathrm{d}s \tag{19}$$

where $\boldsymbol{e}_z(s)$ denotes the Laplace transform of $\boldsymbol{e}_z$, and $s = \sigma + \mathrm{j}w$, which is a complex variable.

The transfer function from $\begin{bmatrix} -\boldsymbol{\xi} \\ \boldsymbol{B}_1\boldsymbol{D}_u \end{bmatrix}$ to $\boldsymbol{e}_z(s)$ is given by

$$\boldsymbol{e}_z(s) = -\boldsymbol{\Omega}_{11}\boldsymbol{\xi} + \boldsymbol{\Omega}_{12}\boldsymbol{B}_1\boldsymbol{D}_u = -\boldsymbol{\Omega}_{11}[\boldsymbol{\xi} - \boldsymbol{B}_z(s\boldsymbol{I} - \boldsymbol{A}_c)^{-1}\boldsymbol{B}_1\boldsymbol{D}_u]$$

This means that optimal J is achieved when

$$\boldsymbol{\xi} = \boldsymbol{B}_z(s\boldsymbol{I} - \boldsymbol{A}_c)^{-1}\boldsymbol{B}_1\boldsymbol{D}_u \tag{20}$$

The state space realization of Eq. (20) is given by

$$\left.\begin{aligned}\dot{\boldsymbol{x}}_{\mathrm{aw}} &= \boldsymbol{A}_c\boldsymbol{x}_{\mathrm{aw}} + \boldsymbol{B}_1[\mathrm{sat}(\boldsymbol{u}) - \boldsymbol{u}] \\ \boldsymbol{\xi} &= \boldsymbol{B}_z\boldsymbol{x}_{\mathrm{aw}}\end{aligned}\right\} \tag{21}$$

Consequently, the proposed dynamic anti-windup controller is given by

$$\left.\begin{aligned}\dot{z} &= \boldsymbol{A}_z z + \boldsymbol{B}_z\boldsymbol{x} - \boldsymbol{B}_z\boldsymbol{x}_{\mathrm{aw}} \\ \dot{\boldsymbol{x}}_{\mathrm{aw}} &= \boldsymbol{A}_c\boldsymbol{x}_{\mathrm{aw}} + \boldsymbol{B}_1[\mathrm{sat}(\boldsymbol{u}) - \boldsymbol{u}] \\ \boldsymbol{u} &= \boldsymbol{K}_d z + (\boldsymbol{K}_x + \boldsymbol{K}_d\boldsymbol{L})\boldsymbol{x}\end{aligned}\right\} \tag{22}$$

2.4 Stability analysis

Given that a static controller with known disturbance $\boldsymbol{u} = \boldsymbol{K}_x\boldsymbol{x} + \boldsymbol{K}_d\boldsymbol{d}$ has been pre-designed to make the constrained system locally asymptotically stable with a domain of attraction, in this section, we address the stability of the closed-loop system under the modified control with unknown disturbance. The stability proof is inspired by Yoon et al. [17] and Kapoor et al[18]. To achieve this, we first introduce the following lemma.

Lemma 1 Here, consider the following cascade system:

$$\left.\begin{aligned}\dot{z}_1 &= \boldsymbol{A}_1 z_1 \\ \dot{z}_2 &= f(z_2) + g(z_2)\mathrm{sat}(k_1(z_2) + k_2(z_2)z_1)\end{aligned}\right\} \tag{23}$$

where $z_1 \in \mathbf{R}^{n_1}$ and $z_2 \in \mathbf{R}^{n_2}$ are the states of system (Eq. (23)); matrix $\boldsymbol{A}_1$ is Hurwitz; functions $f(\cdot)$, $g(\cdot)$, $k_1(\cdot)$ and $k_2(\cdot)$ are given and locally Lipschitz, and $z_2 = \boldsymbol{O}$ is a locally asymptotically stable state for the following system:

$$\dot{z}_2 = f(z_2) + g(z_2)\mathrm{sat}(k_1(z_2)) \tag{24}$$

with a domain of attraction $\boldsymbol{X}_2 \subset \mathbf{R}^{n_2}$, and sat$(\cdot)$ is globally Lipschitz and bounded. Then, given any compact subset $\boldsymbol{Z}_2$ of $\boldsymbol{X}_2$, the equilibrium of the system (Eq. (23)) is locally asymptotically stable with a domain of attraction $\boldsymbol{S}_1 \times \boldsymbol{S}_2$, where $\boldsymbol{S}_1$ is a subset characterized by matrix $\boldsymbol{A}_1$ or $\boldsymbol{X}_2$ equivalently, and set $\boldsymbol{S}_2$ is a compact subset of $\boldsymbol{X}_2$ containing $\boldsymbol{Z}_2$.

For a complete proof of Lemma 1, see the work of Yoon et al. [17] and Kapoor et al[18]. The basic idea is that z_2 remains within a compact subset of $\boldsymbol{X}_2$ as z_1 decays to the origin exponentially. This is achieved as follows. The second equation of the system (Eq. (23)) can be written in an equivalent form as

$$\begin{aligned}\dot{z}_2 = f(z_2) + g(z_2)\mathrm{sat}(k_1(z_2)) + \\ g(z_2)[\mathrm{sat}(k_1(z_2) + k_2(z_2)z_1) - \mathrm{sat}(k_1(z_2))]\end{aligned} \tag{25}$$

where $\boldsymbol{\eta}_{\mathrm{pert}} = g(z_2)[\mathrm{sat}(k_1(z_2) + k_2(z_2)z_1) - \mathrm{sat}(k_1(z_2))]$.

First, it can be proven that the term $\boldsymbol{\eta}_{\mathrm{pert}}$ can be made arbitrarily small (e.g., $\|\boldsymbol{\eta}_{\mathrm{pert}}\| < \varepsilon_z$) in a given time $t = \tau$ for z_2 in a bounded area $\boldsymbol{Z}_2$ with an appropriately selected bounded area $\boldsymbol{S}_1$ for z_1, because function sat$(\cdot)$ is globally Lipschitz and bounded and z_1 is exponentially stable. Then, the nominal system (Eq. (24)) (i.e., system (Eq. (25)) without perturbation) is locally asymptotically stable with a domain of attraction $\boldsymbol{X}_2$. Consequently, for states z_2 in $\boldsymbol{Z}_2 \in \boldsymbol{X}_2$, only perturbation with sufficient amplitude and sufficient time can drive its states out of $\boldsymbol{Z}_2$. Therefore, bound ε_z can be chosen sufficiently small such that the system (Eq. (25)) is asymptotically stable.

Next, we formulate the closed-loop systems with actuator saturation (Eq. (12)) and modified DOBC (Eq. (22)) such that Lemma 1 can be applied.

By defining $\boldsymbol{x}_{\mathrm{T}} = \boldsymbol{x} - \boldsymbol{x}_{\mathrm{aw}}$, we obtain its dynamic as follows:

$$\begin{aligned}\dot{\boldsymbol{x}}_{\mathrm{T}} = \dot{\boldsymbol{x}} - \dot{\boldsymbol{x}}_{\mathrm{aw}} = \boldsymbol{A}_c\boldsymbol{x} + \boldsymbol{B}_c z + \boldsymbol{B}_2\boldsymbol{d} + \boldsymbol{B}_1\boldsymbol{D}_{\boldsymbol{u}} - \\ \boldsymbol{A}_c\boldsymbol{x}_{\mathrm{aw}} - \boldsymbol{B}_1\boldsymbol{D}_{\boldsymbol{u}} = \boldsymbol{A}_c\boldsymbol{x}_{\mathrm{T}} + \boldsymbol{B}_c z + \boldsymbol{B}_2\boldsymbol{d}\end{aligned}$$

Similarly, by defining $\tilde{z}_1 = \begin{bmatrix} z \\ \boldsymbol{x}_{\mathrm{T}} \end{bmatrix}$, we obtain its dynamic as

$$\begin{bmatrix} \dot{z} \\ \dot{\boldsymbol{x}}_{\mathrm{T}} \end{bmatrix} = \begin{bmatrix} \boldsymbol{A}_z & \boldsymbol{B}_z \\ \boldsymbol{B}_c & \boldsymbol{A}_c \end{bmatrix}\begin{bmatrix} z \\ \boldsymbol{x}_{\mathrm{T}} \end{bmatrix} + \begin{bmatrix} \boldsymbol{O} \\ \boldsymbol{B}_2\boldsymbol{d} \end{bmatrix}$$

which is asymptotically stable and converges to its equilibrium $\tilde{z}_1(\infty)$ if $\boldsymbol{d}$ is a constant or has steady state value. By defining $z_1 = \tilde{z}_1 - \tilde{z}_1(\infty)$ such that the equilibrium is shifted to zero, we obtain

$$\dot{z}_1 = \boldsymbol{A}_Y z_1 \tag{26}$$

Furthermore, we consider the following closed-loop system:

$$\begin{aligned}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\mathrm{sat}(\boldsymbol{u}) + \boldsymbol{B}_2\boldsymbol{d} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{d} + \\ \boldsymbol{B}_1\mathrm{sat}(\boldsymbol{K}_d z + (\boldsymbol{K}_x + \boldsymbol{K}_d\boldsymbol{L})\boldsymbol{x})\end{aligned} \tag{27}$$

Using $z_1 = \begin{bmatrix} z \\ \boldsymbol{x}_{\mathrm{T}} \end{bmatrix} - \tilde{z}_1(\infty)$, we obtain

$$z = [\boldsymbol{I} \quad \boldsymbol{O}](z_1 + \tilde{z}_1(\infty)) \tag{28}$$

Substituting Eq. (28) into Eq. (27), we obtain

$$\begin{aligned}\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{d} + \boldsymbol{B}_1\mathrm{sat}((\boldsymbol{K}_x + \boldsymbol{K}_d\boldsymbol{L})\boldsymbol{x} + \\ \boldsymbol{K}_d[\boldsymbol{I} \quad \boldsymbol{O}]\tilde{z}_1(\infty) + \boldsymbol{K}_d[\boldsymbol{I} \quad \boldsymbol{O}]z_1)\end{aligned} \tag{29}$$

where $f(z_2) = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_2\boldsymbol{d}$, $g(z_2) = \boldsymbol{B}_1$, $k_1(z_2) = (\boldsymbol{K}_x + \boldsymbol{K}_d\boldsymbol{L})\boldsymbol{x} + \boldsymbol{K}_d[\boldsymbol{I} \quad \boldsymbol{O}]\tilde{z}_1(\infty)$, $k_2(z_2) = \boldsymbol{K}_d[\boldsymbol{I} \quad \boldsymbol{O}]$.

Eqs. (26) and (29) fall into the same format as Eq. (23) by choosing $z_2 = \boldsymbol{x}$; thus, Lemma 1 can be applied. The results are summarized in the following theorem under the following assumptions.

Assumption 4 Under known $\boldsymbol{d}$, we assume that the system $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}_1\mathrm{sat}(\boldsymbol{u}) + \boldsymbol{B}_2\boldsymbol{d}$ under the controller $\boldsymbol{u} = \boldsymbol{K}_x\boldsymbol{x} + \boldsymbol{K}_d\boldsymbol{d}$ is locally asymptotically stable with a basin of attraction $\boldsymbol{X} \subset \mathbf{R}^n$.

This assumption is the basic design requirement for a system with input saturation.

Assumption 5 The equilibrium of the constrained system $\dot{x} = Ax + B_1 \text{ sat}(u) + B_2 d$ under control $u = K_x x + K_d \hat{d}$ is unique and is the same as that of the unconstrained system $\dot{x} = Ax + B_1 u + B_2 d$ under the same calculated control (u).

This assumption is reasonable, which guarantees that the control amplitude is sufficiently large to reject the disturbances, at least in the steady state.

Theorem 1 Under Assumptions 1-5, if there exists a static feedback control law $u_n = K_x x + K_d d$ for known d, which locally asymptotically stabilizes the saturated system (Eq. (1)) with a domain of attraction X_2, then given any compact subset Z_2 of X_2, the equilibrium of the constrained system (Eq. (1)) under the proposed DO-based anti-windup controller (Eq. (22)) is locally and asymptotically stable with a domain of attraction $S_1 \times S_2$. $S_1 \subset \mathbf{R}^{n_z + n_{aw}}$ is a subset characterized by constant matrix A_Y or X_2 equivalently. Subset S_2 is a compact subset of X_2 containing Z_2.

Proof Eqs. (26) and (29) follow the same format as Eq. (23) of Lemma 1. Therefore, the proof can be completed using Lemma 1.

3 Case Study

A numerical example is presented to demonstrate the main results. Consider a mass, spring, and damper system depicted in Fig. 2. The dynamic can be modeled based on physical law as follows:

$$m\ddot{x} + b\dot{x} + kx = F$$

where the nominal values of mass m, damper coefficient b, and spring constant k are 1 kg, 20 N · s/m, and 10 N/m, respectively. To define $X = [x \; \dot{x} \; \int (x - r)\mathrm{d}t]^{\mathrm{T}}$ with r being the reference signal, $u = F$, we can derive a state space model for the system as

$$\dot{X} = \begin{bmatrix} 0 & 1 & 0 \\ -\frac{k}{m} & -\frac{b}{m} & 0 \\ 1 & 0 & 0 \end{bmatrix} X + \begin{bmatrix} 0 \\ \frac{1}{m} \\ 0 \end{bmatrix} (\text{sat}(u) + d) + \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} r$$

where $A = \begin{bmatrix} 0 & 1 & 0 \\ -k/m & -b/m & 0 \\ 1 & 0 & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1/m \\ 0 \end{bmatrix}$, $B_r = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$; sat($u$) reflects the actuator saturation. The control objective is to track reference signal r while rejecting the adverse effect of d with saturated control sat(u).

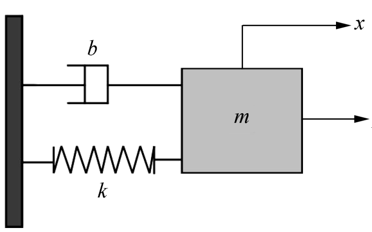

Fig. 2 Diagram of a mass, spring, and damper system

In this study, a nominal DOBC (Eq. (5)) without considering control saturation is pre-designed to satisfy the nominal performance and specification. The state feedback matrix in Eq. (5) for state regulation is designed as $K_x = [-45 \quad 1 \quad -30]$. On the contrary, the disturbance rejection control gain matrix is designed as $K_d = -1$ because disturbance d satisfies the matching condition[2]. The disturbance observer gain matrix is designed as $L = [0 \quad 10 \quad 0]$ such that $-LB_2$ is Hurwitz. To mimic actuator saturation, the upper and lower bounds of the controller are set to be ± 36. The reference position is given by the following piecewise function:

$$r(t) = \begin{cases} 1 & 0 \leqslant t \leqslant 12 \text{ s} \\ -1 & 12 \text{ s} < t \leqslant 20 \text{ s} \\ 0 & 20 \text{ s} < t \leqslant 30 \text{ s} \end{cases}$$

In the following subsections, comparative simulations are performed to evaluate the three controllers, including DOBC without actuator saturation (DOBC-USAT), DOBC with actuator saturation (DOBC-SAT), and the proposed modified DOBC with anti-windup (DOBC-ANTI) for external disturbances and parameter perturbations.

3.1 External disturbances

We first compare the performance of the three controllers under external disturbances. An unknown step disturbance with an amplitude of 45 N is imposed on the system during 4-8 s. Fig. 3 shows the results under different controllers, including position trajectories, control inputs, distribution estimations, and z-dynamics.

Fig. 3(a) shows the following observations. First, without actuator saturation, DOBC obtains satisfactory performance for reference tracking and disturbance rejection. Second, under actuator saturation, control performance will degrade (i. e., larger overshoot or longer setting time). Third, DOBC-ANTI can substantially improve the performance of DOBC-SAT. The root mean squared errors (RMSEs) of the three controllers' position trajectories are 0.215 2, 0.284 8, and 0.262 6, respectively. As shown in Fig. 3(c), DOBC-SAT fails to obtain external disturbance information due to actuator saturation, whereas the disturbance estimation of DOBC-ANTI is close to that of DOBC-USAT. The RMSEs of the three controllers' disturbance estimation (DOBC-USAT, DOBC-SAT, and DOBC-ANTI) are 2.624 0, 57.976 6, and 3.286 0, respectively. As shown in Fig. 3(d), the

z-dynamic of DOBC-ANTI is the same as that of DOBC-USAT, which means that the effect of actuator saturation on the z-dynamic has been removed by designing an anti-windup compensator.

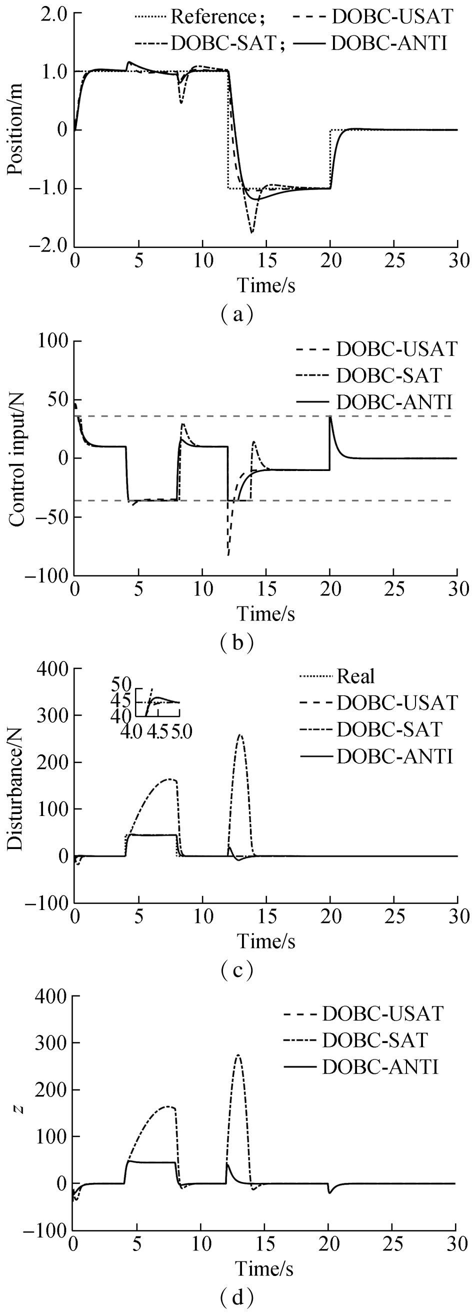

Fig. 3 Results for external disturbances. (a) Reference tracking; (b) Control input; (c) Disturbance estimation; (d) z-dynamic

3.2 Parameter perturbations

The effect of parameter perturbations on different controllers is further compared. In the following simulation, the system parameters are chosen as $m = 1.4m_0$, $k = 0.8k_0$, and $b = 1.3b_0$, with m_0, k_0, and b_0 being the nominal values. Fig. 4 shows the comparative results under this scenario.

Figs. 4(a)-(c) show that DOBC can obtain satisfactory

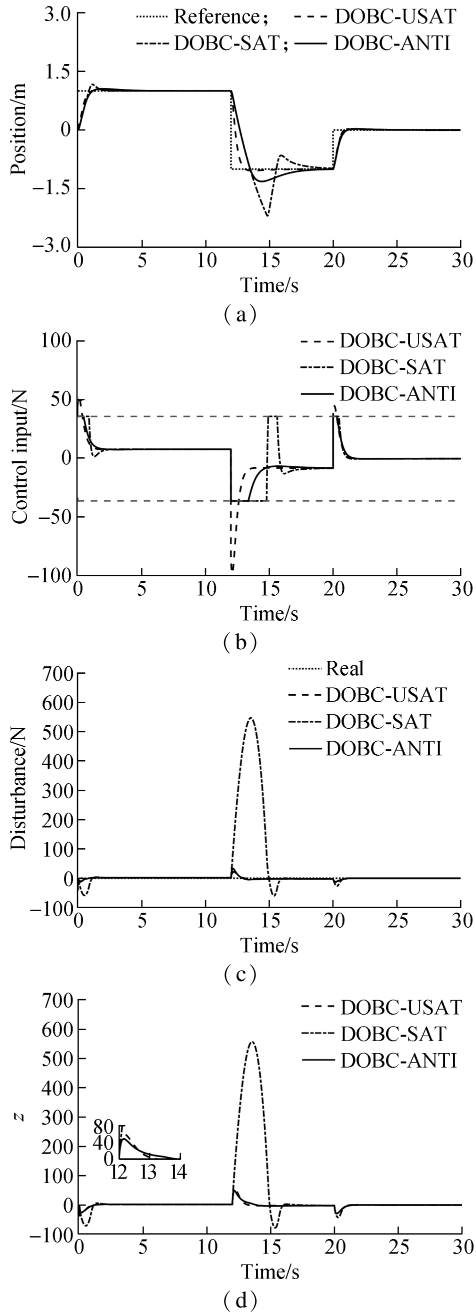

Fig. 4 Results for parameter perturbations. (a) Reference tracking; (b) Control input; (c) Disturbance estimation; (d) z-dynamic

performance without actuator saturation. However, the performance will degrade during actuator saturation because the disturbance estimation is unrealistic and results in windup. The proposed anti-windup strategy can mitigate the effect of actuator saturation. Specifically, the RMSEs of the three controllers' (DOBC-USAT, DOBC-SAT, and DOBC-ANTI) position trajectories are 0.219 0, 0.351 7, and 0.298 8, respectively, and the RMSEs of their disturbance estimation are 3.417 7, 123.674 8, and 4.201 2, respectively. As shown in Fig. 4(d), the z-dynamics of DOBC-USAT and DOBC-ANTI possess a subtle difference due to parameter uncertainties.

4 Conclusions

1) DOBC without actuator saturation is designed to counteract the effects of unknown disturbances/uncertainties. A compensator is also designed and patched into the existing disturbance observer, resulting in a modified DOBC with anti-windup ability.

2) Solid stability analysis is performed on the proposed control inspired by the existing results in the field of anti-windup control.

3) An academic example is provided to demonstrate the superiority of the proposed control for disturbance rejection and anti-windup. Moreover, the designed compensator can maintain the DO states to be as close as possible to those without actuator saturation.

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具有执行器饱和的不确定系统的干扰抑制控制

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摘要:针对传统的干扰观测器(DO)设计并未考虑实际系统中普遍存在的执行器饱和问题,提出了一种具有抗饱和补偿的基于干扰观测器的控制(DOBC)策略. 首先探究传统 DOBC 在执行器饱和时产生积分饱和现象的原因;然后通过最小化性能指标设计了一种抗积分饱和的补偿器,并将其与 DO 结合,使得修改后的 DOBC 能够有效处理执行器饱和;最后对所得闭环系统进行局部渐近稳定性分析. 比较仿真结果表明,存在执行器饱和时,所提出控制方法具有更小的位置跟踪误差和干扰估计误差,并且设计的补偿器可以保持 DO 的状态尽可能接近没有执行器饱和时的状态,进而验证了所提出控制方法在抗干扰和抗饱和方面具有优越性.

关键词:抗积分饱和;干扰观测器;动态补偿器;执行器饱和

中图分类号:TP13