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Cable force optimization of cable-stayed bridges based on the influence matrix and elite genetic algorithm

Zhu Li[1] Meng Bowen[1] Huo Xuejin[2] Liu Wei[1]

([1]School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, China)

([2]China Railway Major Bridge Reconnaissance and Design Institute Co., Ltd., Wuhan 430050, China)

Abstract: The limitations of conventional cable force optimization methods, which fail to automatically optimize and consider the overall performance of the bridge structure, as well as the drawbacks of extensive calculations, lengthy processing time, low efficiency, and slow convergence speed, when combined with intelligent optimization algorithms, should be addressed. Ansys and Matlab are used as the structural calculator and master control programs, respectively, with the minimum bending moment energy as the control objective. Moreover, the influence matrix and elite retention strategy are incorporated into the genetic algorithm to optimize the cable force during the bridge formation stage. This method can simultaneously account for the force characteristics of the main girder and pylon. Utilizing the influence matrix, the issue that each generation requires finite element evaluation can be resolved, thereby drastically reducing the amount of calculation. In addition to capitalizing on the benefits of the conventional influence matrix method, the proposed approach considers the iterative process of parameter selection and permits the addition of special constraint requirements to critical sections of the structure, thereby enhancing the realism of the optimization procedure. Furthermore, the introduction of the elite retention strategy enhances the convergence speed and stability of evolutionary iterations. Finally, a practical engineering application is utilized to validate the viability of the proposed method.

Key words: cable-stayed bridge; cable force optimization; minimum bending moment energy; influence matrix; genetic algorithm; feasible domain

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Decades of research and development have resulted in the introduction of new materials and technologies that effectively increase the spanning capacity of bridge structures, in addition to the relatively lightweight and esthetically pleasing structures of modern cable-stayed bridges[1]. Hence, the material efficiency needs to be enhanced, and the costs need to be reduced through a more precise structural design that optimizes the overall structure. The structural force is substantially influenced by the cable force, which is a critical force transmission element in a cable-stayed bridge. Consequently, during the bridge formation phase, the cable force must be precisely optimized for the cable-stayed bridge. Wang et al.[2] established the zero displacement method to achieve the minimum displacement of the primary girder. Focusing solely on displacement as the objective can result in excessive bending moments at specific locations. Chen et al.[3] proposed the internal force balance method as a solution to the problems of the zero displacement method. Fan et al.[4] proposed using the sum of structural bending moments squared as the optimization target. The minimum bending energy method, which was developed by Zhou et al.[5], circumvents the shortcomings of the sum of squares method by incorporating the weighting effects of the bending rigidity of the main girder and pylon into the bending moment. Liang et al.[6] presented a workable approach that is quick and easy to use. This method can be considered a condensed form of the influence matrix technique. Xiao et al.[7] introduced the influence matrix method, which effectively manages and modifies the internal forces and displacements on the crucial sections of the main girder and main tower. Huang et al.[8] integrated the rigid boom and influence matrix methods to optimize the cable force. Li et al.[9] determined that the influence matrix method, in conjunction with the rigidly supported continuous beam method, could produce the ideal state of the completed bridge more rapidly and efficiently. However, the continuous development of computers has significantly enhanced computing power, which has brought new ideas for the optimization of cable force. Atmaca et al.[10–11] combined the metaheuristic algorithm Jaya with the optimization procedure. The cable force was optimized by considering the cable stress and main beam displacement. The result was a cable force that was suitable for the cable. Wang et al.[12] and Dan et al.[13] employed a multi-objective particle swarm optimization approach, in conjunction with the influence matrix method, to develop the mathematical model for optimizing the cable force. The method reduced the optimization time

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Biography: Zhu Li (1986—), male, doctor, professor, zhuli@bjtu.edu.cn.

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and improved the optimization results. Song et al.[14-15] emphasized that, for asymmetric cable-stayed bridges, a counterweight on the side span must be implemented. A novel alternative model-assisted differential evolution method is proposed to enhance the power of cable force optimization. Sung et al.[16] investigated the incorporation of simulated annealing and particle swarm optimization into the mutation of the genetic algorithm, which enhanced the capacity of the algorithm to break out from the local optimal solution. Ha et al.[17] proposed a cable force optimization approach based on the nonlinear inelastic analysis and micro-genetic algorithm. In addition to accounting for the cable sag effect, first-order effect, and large displacement, this method significantly reduces the amount of calculation needed. A novel algorithm that incorporates simulated annealing algorithms and cubic spline interpolation curves was introduced by Guo et al[18-19]. The optimization technique is enhanced by considering the counterweight problem, cable droop problem, and geometric nonlinearity, ensuring that the approach is more applicable to real-world scenarios.

The traditional optimization methods need to manually adjust the finite element model, which is unable to complete the automation of the optimization process and has the disadvantage of a cumbersome and time-consuming optimization process. By integrating advanced mathematical optimization techniques with finite element analysis, the finite element evaluation of each generation during the iteration process needs to be conducted. This approach significantly increases the computational workload, resulting in time-consuming and inefficient outcomes. Thus, a novel software application that can accurately ascertain the most favorable cable force for such bridges needs to be devised. Taking the minimum bending moment energy as the target, the stress characteristics of the main beam and main tower can be considered at the same time. Therefore, the minimum bending moment energy is used as the target variable, and the influence matrix method is implemented to derive the explicit expression of the optimization objective function. Integrating the influence matrix and genetic algorithm may effectively address the limitations of finite element evaluations in every generation, minimize the number of computations, and boost the optimization speed. The optimization phase of the genetic algorithm incorporates the elite retention strategy[20], which enhances the convergence speed and stability of the evolution process. To validate the feasibility and accuracy of the method, an engineering example of a large-span cable-stayed bridge is utilized.

1 Description of the Optimization Problem

During the design phase of a cable-stayed bridge, material utilization, which is highest when there is axial force, needs to be considered. Once the structure is determined, the girder and pylon of the bridge need to be optimized to maximize the axial force by minimizing the bending energy. The overall structural safety factor of the cable-stayed bridge is currently elevated. By optimizing the cable force to minimize the bending moment energy, both the main pylon and main girder can be simultaneously considered. This approach is beneficial for achieving the optimal distribution of the bending moment in the tower girder. Hence, the minimum bending moment energy is designated as the target variable in this study.

1.1 Determination of the bending strain energy

The objective function of this study is to minimize the bending moment energy. The bending moment energy expression of the structure is represented by the influence matrix and bending moment vectors of the primary girder and pylon[21]. The equation for the expression of the bending moment energy is written as follows:

$$U = \int_s \frac{M^2(s)}{2EI}\mathrm{d}s \tag{1}$$

where U is the bending moment strain energy of the structure; $M(s)$ is the bending moment of the unit; E and I are the modulus of elasticity of the material and the moment of inertia of the cross-section of the structure, respectively.

Given that the finite element model represents discrete rod constructions for the main girder and main pylon, Eq. (1) can be written as follows:

$$U = \sum_{i=1}^{m} \frac{l_i}{4E_iI_i}(M_{\mathrm{L}i}^2 + M_{\mathrm{R}i}^2) \tag{2}$$

where m is the number of units of the pylon and girder divided in the finite element software; l_i, E_i, and I_i are the length, material elastic modulus, and cross-sectional moment of inertia of the main girder and i-th main pylon unit i, respectively; and $M_{\mathrm{L}i}$ and $M_{\mathrm{R}i}$ are the bending moments at the left and right ends of the i-th beam unit, respectively. Eq. (2) can be rewritten in terms of matrix multiplication and addition as follows:

$$U = \boldsymbol{M}_{\mathrm{L}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{M}_{\mathrm{L}} + \boldsymbol{M}_{\mathrm{R}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{M}_{\mathrm{R}} \tag{3}$$

where $\boldsymbol{M}_{\mathrm{L}}$ and $\boldsymbol{M}_{\mathrm{R}}$ are the left and right end bending moment vectors, respectively, of all girder units, and $\boldsymbol{B}$ is the coefficient matrix of the bridge structure.

$$\boldsymbol{B} = \begin{bmatrix} b_{11} & & & \\ & b_{22} & & \\ & & \ddots & \\ & & & b_{mm} \end{bmatrix}$$

$$b_{ii} = \frac{l_i}{4E_iI_i} \qquad i = 1, 2, \ldots, m \tag{4}$$

where b_{ii} is the bridge structural coefficient for different units of the main girder and main pylon. Assuming that the bending moment vectors at the ends of the main girder and main tower in the initial state are $\boldsymbol{M}_{\mathrm{L0}}$ and $\boldsymbol{M}_{\mathrm{R0}}$, respectively, the cable force needs to be adjusted to achieve the goal of the minimum bending moment energy, and the adjustment vector of the cable force is set as $\boldsymbol{T}$. Then, the bending moment vectors at the two ends of the girder unit after the adjustment can be expressed as follows:

$$\left.\begin{aligned}\boldsymbol{M}_{\mathrm{L}} &= \boldsymbol{M}_{\mathrm{L0}} + \boldsymbol{C}_{\mathrm{L}}\boldsymbol{T}\boldsymbol{M}_{\mathrm{L}}\\ \boldsymbol{M}_{\mathrm{R}} &= \boldsymbol{M}_{\mathrm{R0}} + \boldsymbol{C}_{\mathrm{R}}\boldsymbol{T}\boldsymbol{M}_{\mathrm{R}}\end{aligned}\right\} \tag{5}$$

where $\boldsymbol{C}_{\mathrm{L}}$ and $\boldsymbol{C}_{\mathrm{R}}$ are the influence matrix of the cable force on the left and right end moments of the girder units in a cable-stayed bridge, respectively. The specific bending moment energy expression can be derived by substituting the bending moment vectors at the extremities of the beam unit following the cable force adjustment to the bending moment energy expression as follows:

$$\begin{aligned}U = C_0 + \boldsymbol{M}_{\mathrm{L0}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{C}_{\mathrm{L}}\boldsymbol{T} + \boldsymbol{T}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{L}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{M}_{\mathrm{L0}} + \boldsymbol{T}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{L}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{C}_{\mathrm{L}}\boldsymbol{T} +\\ \boldsymbol{M}_{\mathrm{R0}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{C}_{\mathrm{R}}\boldsymbol{T} + \boldsymbol{T}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{R}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{M}_{\mathrm{R0}} + \boldsymbol{T}^{\mathrm{T}}\boldsymbol{C}_{\mathrm{R}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{C}_{\mathrm{R}}\boldsymbol{T}\end{aligned} \tag{6}$$

C_0 in Eq. (6) can be considered the bending moment energy present in the initial bridge structure and is expressed as follows:

$$C_0 = \boldsymbol{M}_{\mathrm{L0}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{M}_{\mathrm{L0}} + \boldsymbol{M}_{\mathrm{R0}}^{\mathrm{T}}\boldsymbol{B}\boldsymbol{M}_{\mathrm{R0}} \tag{7}$$

1.2 Access to the influence matrix

To derive the influence matrices $\boldsymbol{C}_{\mathrm{R}}$ and $\boldsymbol{C}_{\mathrm{L}}$ of cable forces on the bending moments at the extremities of the primary girder and pylon components, we employ a straightforward single-tower cable-stayed bridge, as illustrated in Fig. 1. The main girder and pylon consist of nine girder parts, labeled a, b, …, i. Four wires are positioned diagonally, specifically labeled ①, ②, ③, and ④, respectively. Under the starting conditions, when all four cables have no force, the bending moment vectors at both ends of the beam element are $\boldsymbol{D}_{\mathrm{L0}}$ and $\boldsymbol{D}_{\mathrm{R0}}$. The adjustment vector represents the force in the diagonal cable, while the adjusted vector represents the bending moment at both ends of the beam element. A unit force is applied to Cable ①, while the other cable forces are maintained at zero. At this point, the bending moment vectors at both ends of the beam element are represented by $\boldsymbol{D}_{\mathrm{L1}}$ and $\boldsymbol{D}_{\mathrm{R1}}$, whereas the influence vector is written as follows:

$$\left.\begin{aligned}\Delta\boldsymbol{D}_{\mathrm{L1}} &= \boldsymbol{D}_{\mathrm{L1}} - \boldsymbol{D}_{\mathrm{L0}}\\ \Delta\boldsymbol{D}_{\mathrm{R1}} &= \boldsymbol{D}_{\mathrm{R1}} - \boldsymbol{D}_{\mathrm{R0}}\end{aligned}\right\} \tag{8}$$

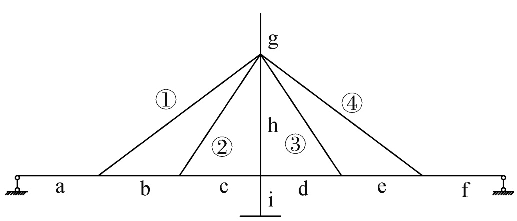


Fig. 1 Single-tower cable-stayed bridge

Similarly, the following expressions can be inferred:

$$\left.\begin{aligned}\Delta\boldsymbol{D}_{\mathrm{L}i} &= \boldsymbol{D}_{\mathrm{L}i} - \boldsymbol{D}_{\mathrm{L0}}\\ \Delta\boldsymbol{D}_{\mathrm{R}i} &= \boldsymbol{D}_{\mathrm{R}i} - \boldsymbol{D}_{\mathrm{R0}}\end{aligned}\right\} \quad i = 1, 2, 3, 4 \tag{9}$$

$$\left.\begin{aligned}&\Delta\boldsymbol{D}_{\mathrm{L}i} =\\ &[d_{\mathrm{L1}i} \quad d_{\mathrm{L2i}} \quad d_{\mathrm{L3}i} \quad d_{\mathrm{L4}i} \quad d_{\mathrm{L5}i} \quad d_{\mathrm{L6}i} \quad d_{\mathrm{L7}i} \quad d_{\mathrm{L8}i} \quad d_{\mathrm{L9}i}]^{\mathrm{T}}\\ &\Delta\boldsymbol{D}_{\mathrm{R}i} =\\ &[d_{\mathrm{R1}i} \quad d_{\mathrm{R2}i} \quad d_{\mathrm{R3}i} \quad d_{\mathrm{R4}i} \quad d_{\mathrm{R5}i} \quad d_{\mathrm{R6}i} \quad d_{\mathrm{R7}i} \quad d_{\mathrm{R8}i} \quad d_{\mathrm{R9}i}]^{\mathrm{T}}\end{aligned}\right\} \tag{10}$$

where $\Delta\boldsymbol{D}_{\mathrm{L}i}$ and $\Delta\boldsymbol{D}_{\mathrm{R}i}$ are the vectors that indicate the bending moment effect of the i-th cable on the left and right ends of the unit, respectively. Similarly, $d_{\mathrm{L}ji}$ and $d_{\mathrm{R}ji}$ are the values of the bending moment influence of the i-th cable on the left and right ends of the j-th unit, respectively.

Assuming a total of m elements in the girder and pylon and n diagonal cables, the force of each diagonal cable increases by one unit. Instances $\boldsymbol{C}_{\mathrm{L}}$ and $\boldsymbol{C}_{\mathrm{R}}$ illustrate the influence matrix $\boldsymbol{C}_{\mathrm{L}}$, which is the product of the influence vectors produced by m elements.

$$\boldsymbol{C}_{\mathrm{L}} = [\Delta\boldsymbol{D}_{\mathrm{L1}} \quad \Delta\boldsymbol{D}_{\mathrm{L2}} \quad \cdots \quad \Delta\boldsymbol{D}_{\mathrm{L}n}] = \begin{bmatrix} d_{\mathrm{L11}} & d_{\mathrm{L12}} & \cdots & d_{\mathrm{L1}n}\\ d_{\mathrm{L21}} & d_{\mathrm{L22}} & \cdots & d_{\mathrm{L2}n}\\ \vdots & \vdots & & \vdots\\ d_{\mathrm{L}m1} & d_{\mathrm{L}m1} & \cdots & d_{\mathrm{L}mn}\end{bmatrix} \tag{11}$$

In the context of the current project, the influence of each cable on the vertical bending moment of the main girder under unit cable force is depicted in Fig. 2, where the influence of a cable tends to diminish with increasing distance from the anchor point of the stay cable, which is generally consistent with the actual situation.

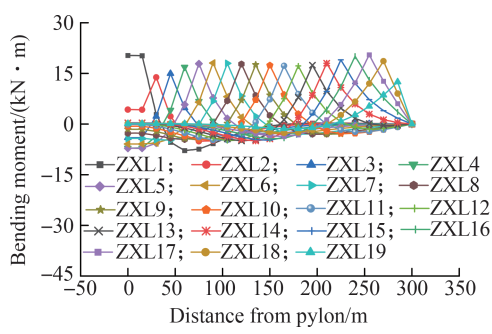


Fig. 2 Influence of the tension cable forces on the vertical bending moment of the main girder

2 Genetic Algorithm

The genetic algorithm was first developed by Holland[22] in 1975 and was subsequently condensed into an intelligent optimization method by De[23] and Goldberg[24]. The optimized calculation method of the genetic algorithm emulates the natural evolution process of biological genes, including inheritance, mutation, and crossover. This approach has bionic properties, has a robust capability to search globally, and is not instance-dependent. Hence, the genetic algorithm compensates for the lack of knowledge regarding the exact range of the lowest bending moment energy and has significant practical significance in optimizing cable forces for cable-stayed bridges. In the population reproduction space, Fig. 3 illustrates the process of continuous evolution and optimization of elite individuals. The global optimal solution in its ideal state is denoted by Z. Q_{best} is the historical optimal solution on a global scale, while J_{best} is the local optimal solution. E_{L} is the elite of the initial population, whereas O_{R} is the average person. Individuals have higher individual fitness values as their positions increase.

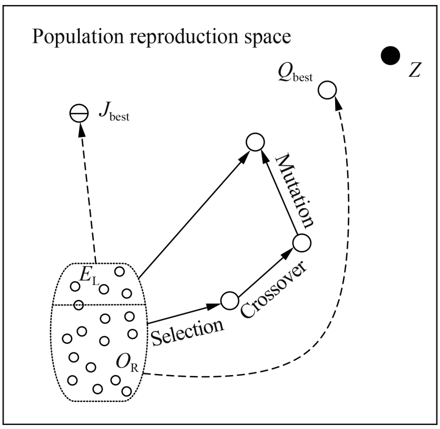


Fig. 3 Process of population reproduction optimization

2.1 Modeling analysis

The Ansys software is used to create a finite element model of a cable-stayed bridge. The particular guidelines and procedures used are as follows: The primary control program utilized is Matlab. Finite element model analysis is performed by invoking Ansys from Matlab. By obtaining the influence matrix indicated previously, a suitable cable force increment is applied based on the starting cable force, and the influence matrix can be obtained by calling Ansys constantly via Matlab.

2.2 Generating populations

A population of p_{s} pieces, representing a collection of cable forces, is generated at random within a specified range using Matlab. Binary codes are initially generated to simulate the genetic sequence of the gene. The process of converting decimal numbers into binary numbers and binary numbers into decimal numbers is referred to as encoding and decoding, respectively. The utilization of binary information facilitates individual selection, crossover, and mutation. The binary code must be translated into a decimal value before it can be inputted into Ansys. This study determined that a small population size would lead to a large bias in the final optimization results. The initial population size should be set moderately.

2.3 Transforming the objective function into a fitness function

Eq. (6) illustrates the bending strain energy of the main girder and main structure of the cable-stayed bridge, which serves as the objective function of the genetic algorithm described in this article. The fitness function value measures the adaptability of an individual in the population. A higher fitness function value indicates a smaller overall bending strain energy sum, which signifies a stronger individual with a more reasonable cable-staying force. Combining the aforementioned characteristics, the fitness function of this study is expressed as follows:

$$f_i = \frac{1}{o_i} \qquad i = 1, 2, \ldots, p_{\mathrm{s}} \tag{12}$$

where o_i is the total bending strain energy of the main girder and main tower for the case of the i-th set of cable forces, i. e., U as described previously, and f_i is the fitness function value corresponding to an individual of the i-th population.

2.4 Selection, crossover, and mutation

To increase the convergence and global convergence speeds of genetic algorithms, an elite retention strategy is presented in this study. This strategy maintains that individuals with fitness function values within the top $p\%$ of the population are directly retained for the next generation, while the remaining $(1-p)\%$ of the population uses a roulette wheel strategy for the selection and replication operations (or the direct removal of ordinary individuals and replication of elite individuals to reach a population size of p_{s}). The following are the key steps in the roulette wheel strategy.

1) The probability of selection for each individual in the remaining $(1-p)\%$ of the population is calculated as follows:

$$\boldsymbol{P}(i, 1) = \frac{f(i)}{\sum_{i=1}^{n} f(i)} \qquad i = 1, 2, \ldots, n \tag{13}$$

where $\boldsymbol{P}$ is the probability vector selected by the individual; $f(i)$ is the fitness function value of the i-th individual; and n is the total number of individuals in the remaining $(1-p)\%$ of the population.

2) n values between $[0, 1]$ are randomly generated by the rand function in Matlab, and the generated random numbers are subsequently arranged in ascending order $\boldsymbol{R} = \mathrm{sort}(\mathrm{rand}(p_{\mathrm{s}}, 1))$ if

$$\sum_{i=1}^{k} \boldsymbol{P}(i, 1) \geqslant \boldsymbol{R}(i, 1) \tag{14}$$

then the k-th individual is retained, and the selection is finalized by repeating this method n times.

In this study, a fixed probability of crossover and mutation operators is provided for each generation of the generated population to prevent the cable force optimization results from settling into the local optimal solution, as opposed to the conventional genetic algorithm, which does not perform crossover operations every few generations. The procedure for the crossover operator is illustrated in Fig. 4; however, only a schematic diagram is shown. The crossover probability and the position of the crossover point are determined and fine-tuned based on unique requirements.

Fig. 4 Crossover operation

The mutation operator emulates the natural process of DNA mutation, which can lead to specific individuals diverging from the original population and introducing new genetic variations, preventing the optimization results of the algorithm from being trapped in the local optima. The desired probability and random mutation locations for interchanging gene codes "0" and "1" are specified. Fig. 5 depicts the specific procedure.

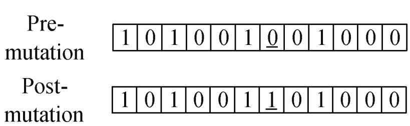


Fig. 5 Mutation operation

2.5 Renewing populations and breeding for excellence

After selection, crossover, and mutation, the new population and individuals are evaluated, the fitness value of each individual is calculated, the individual with the highest fitness value is recorded, and whether the conditions for terminating the iteration are met is checked. If the conditions are met, then the iteration output process is terminated. However, if the conditions are not met, we need to go back to Section 2.3.

3 Specific Process of the Optimization Method

The step-by-step procedure for the cable force optimization method described in this study is illustrated in Fig. 6. The entire procedure is separated into two frameworks, as previously stated. To conduct data interactions and obtain the influence matrix of the finite element model, Ansys is invoked, while Matlab serves as the primary control program. The cable force is subsequently optimized in Matlab using the elite genetic algorithm.

Two optimization strategies are considered in this study. The first optimization strategy involves retaining elite individuals to obtain the next generation by performing crossover, selection, and mutation operations on common individuals in the original population. The second optimization strategy entails eliminating commoners, duplicating the chosen elites to the same number as the initial population, and generating the next generation through the selection, crossover, and mutation processes. In the context of the current project, Fig. 7 illustrates that, during the initial 100 iterations of both approaches,

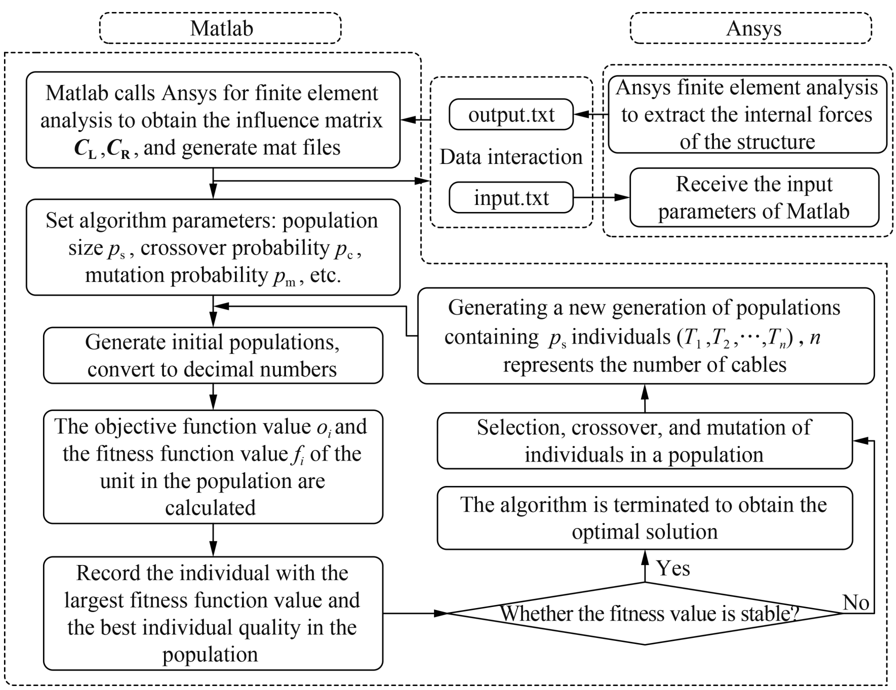


Fig. 6 Optimized program flowchart

the fitness value of the objective function significantly increases. After 200 iterations, the fitness gradually varies and subsequently converges. Notably, the mutation operation must occur before the crossover operation in the second method. Excessive variation in the fitness values of the initial populations distinguishes the two approaches. The main reason for this difference is that the optimal individuals of the initial population randomly generated by the second method are more dominant, followed by the differences in the feasible region of the cable force. The optimal individual of each generation obtained using Method 1 may be better or worse than the previous generation. The optimal individual of each generation obtained using Method 2 may have a larger optimal individual fitness value than the previous generation until convergence. In the present study, the second method is used to optimize the cable force.

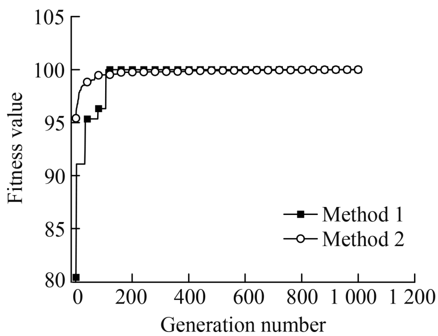


Fig. 7 Fitness value of the objective function

4 Engineering Examples

4.1 Project overview

A large-span cable-stayed bridge is a bridge with a span of 300 m + 300 m, a single tower, and a double cable-stayed design. The bridge is constructed of herringbone girders and utilizes a floating structural system. The primary beam of the cable-stayed bridge is a closed, streamlined, flat steel box beam and has a height of 4.0 m and a width of 35.876 7 m. A cross beam is placed every 5 m. The main beam of this bridge is symmetrical and does not need a counterweight operation. The bridge tower has a rectangular cross-section and has asymmetrical design. Its cross-section is 13.875 3 m long and 7.689 2 m wide across it, and its height is 261.8 m. The diagonal cables consist of planar cables with a harp arrangement. A total of 76 cables are constructed using steel strands with a compressive strength of 1 860 MPa.

The cable-stayed bridge in this study is designed with symmetrical features on both sides. As a result, an additional pier or counterweight operation on the side span is not needed. Applying a live load to both the entire bridge and mid-span is important to the bridge system, as it will result in a more cautious assessment of the cable force. In this study, two distinct types of real load conditions are considered. As shown in Fig. 8(a), the distributed load is 7.875 kN/m, the concentrated load is 270 kN, and a cable-stayed bridge is set up with four lanes. Actual projects should also account for the effects of concrete shrinkage and creep, steel fatigue, temperature, and wind load[25]. In the genetic algorithm, the population size is set as 200, the crossover probability is set as 0.6, and the mutation probability is set as 0.3.

4.2 Establishment of the finite element model of a cable-stayed bridge via Ansys

Based on the fishbone girder model, a finite element model of the cable-stayed bridge is established via Ansys[26]. To simplify the calculation, the main beam and main tower are simulated by the Beam4 unit, and the cable is simulated by the Link10 unit. The section characteristics and material parameters of the model are shown in Table 1. Because the focus of this study is on optimizing the cable force of cable-stayed bridges, the end of the main girder is directly constrained. The model has 921 nodes and 992 elements, as shown in Fig. 8(b).

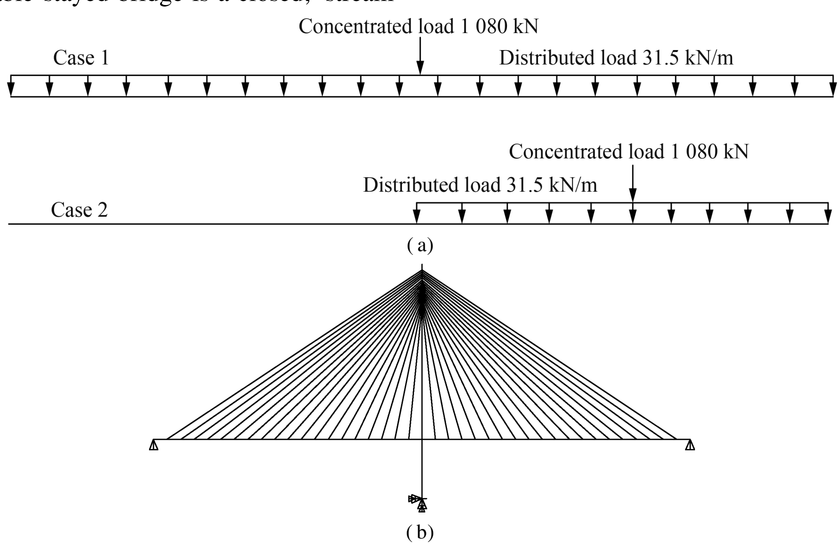


Fig. 8 Ansys finite element model and live load condition diagram. (a) Load distribution; (b) Finite element model

Table 1 Material parameters and section characteristics of the cable-stayed bridge

Structure	I_{yy}/m^4	Area/m^2	Density/(kg · m^{-3})	Poisson's ratio	Elastic modulus/GPa
Girder	230.906	1.768 3	8 005	0.3	206
Pylon	410.989	54.8	2 600	0.2	35.5
Cable		0.012 046	8 005	0.3	195

4.3 Constraint conditions

To ensure the safety of the cable-stayed bridge and improve the utilization rate of the material, the cable force is $0.2f_{\mathrm{ptk}}A_{T_i} < T_i < 0.4f_{\mathrm{ptk}}A_{T_i}$ according to the specification restrictions, where f_{ptk} is the standard value of the tensile strength of the stay cable, and A_{T_i} is the cross-sectional area of the stay cable. For the main beam and main tower, the displacement and internal force should be within a certain range, and the displacement limit of the main beam should be $|S| < 6$ cm. Because the cable-stayed bridge is a left-right symmetrical structure, the displacement effect of the main tower can be disregarded.

To reduce the space of cable force optimization, this study proposes a feasible region of cable force, i. e., the cable force is limited to the set range, as shown in Fig. 9. The four points (L_1, $f_{1\min}$), (L_1, $f_{1\max}$), (L_n, $f_{2\min}$), and (L_n, $f_{2\max}$) constitute the feasible region of cable force, where $f_{1\min}$, $f_{1\max}$ and $f_{2\min}$, and $f_{2\max}$ are the upper and lower limits of the first and last cable force values, respectively, and L is the corresponding position of the anchor point of the cable. The range of the feasible region is related to the optimization speed, and the smaller the range, the faster the cable force optimization. The feasible range of the cable force can be preliminarily determined according to the traditional optimization method, adjusted according to the project requirements, and finally determined.

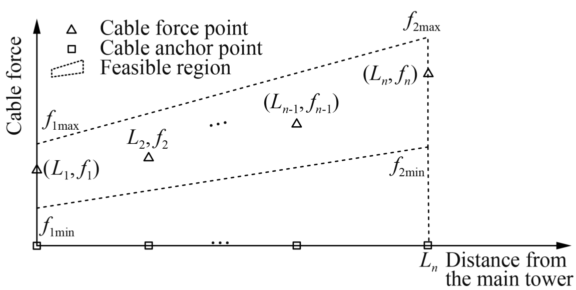


Fig. 9 Feasible region diagram of the cable force

4.4 Optimal results

The entire optimization process in this study takes only 85.6 s. Compared with other optimization methods, the proposed method can not only automatically complete the optimization of the cable force but also rapidly obtain the optimization results so that the optimization efficiency is considerably improved. Fig. 10 illustrates the iterative process of the population within a feasible region of the cable force. The process is optimized for a duration of 200 generations, with each generation consisting of only the best individual, to simplify the presented data. During the early phase of population reproduction iteration, the overall adaptability of the population is low, indicating that it is far from the optimal cable force condition of the bridge. The population rapidly evolves to its ideal state following reproduction and iteration. As a result, the cable force increases from the main tower to the side span, whereas the displacement and bending moment of the main girder remain within an acceptable range. The displacement and bending moment of the main tower invariably satisfy the specifications because of the symmetrical nature of the bridge type. In summary, the optimization outcomes adhere to the tenets of homogeneous cable force, vertical tower, and flat beam optimization principles.

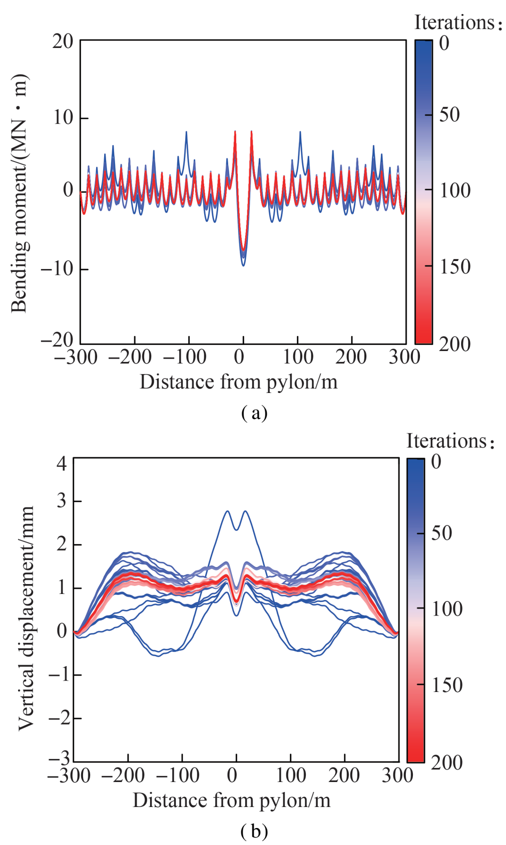


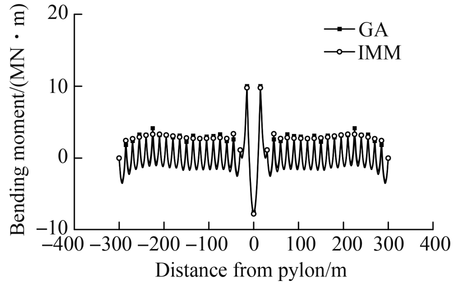
Fig. 10 Girder response and cable force changes within 200 generations. (a) Vertical bending moment of the girder; (b) Vertical displacement of the girder

The optimized cable force values are shown in Table 2, where ZXL1, ZXL2, …, ZXL19 are the stay cable numbers from the main pylon to both sides. After the optimization of the cable force, the influence matrix method is used to optimize the cable force. The maximum cable force is 2 795.90 kN, and the difference between maximum and minimum cable forces is 1 452.25 kN. After introducing the genetic algorithm, the maximum cable force is 2 742.82 kN, and the difference between maximum and minimum cable forces is 1 362.69 kN. A comparison of the maximum and minimum cable force differences reveals that the difference can be controlled within a smaller range after the introduction of the genetic algorithm. The vertical bending moment diagram of the optimized main girder is shown in Fig. 11(a). After the introduction of the genetic algorithm, the maximum negative bending moment of the main girder is 10 012.92 kN · m, and the maximum positive bending moment is 7 524.01 kN · m. The maximum negative bending moment of the influence matrix method is 9 777.75 kN · m, and the maximum positive bending moment is 7 680.81 kN · m. Based on the diagram, the main beam bending moment diagrams of both methods are nearly identical. The main beam bending moment diagram exhibits a uniform oscillation at zero, which closely resembles the ideal condition. Fig. 11(b) shows the vertical displacement of the improved girder. The application of genetic algorithm optimization enables the introduction of a specific precamber to the main beam. In addition, there is a vertical displacement of 1.5 mm in the upward direction at the quarter span. The optimization process of the impact matrix method focuses on minimizing the energy associated with bending moments. As the structure is entirely floating, there will be a vertical displacement of 1.5 mm downward at the main tower site. The aforementioned results show that the proposed method not only has the

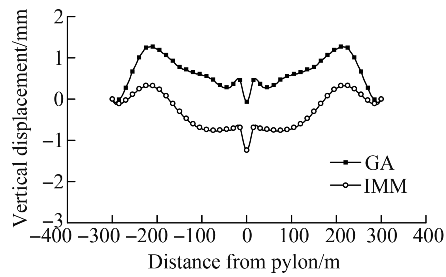
Table 2 Cable force of a cable-stayed bridge at dead load

Cable number	Cable force/kN	
	GA	IMM
ZXL1	2 604.54	2 580.58
ZXL2	1 380.13	1 343.65
ZXL3	1 805.42	1 833.98
ZXL4	1 732.72	1 732.53
ZXL5	1 851.72	1 802.29
ZXL6	1 844.16	1 833.87
ZXL7	1 905.68	1 883.67
ZXL8	1 917.44	1 935.18
ZXL9	2 058.74	1 992.10
ZXL10	1 987.06	2 052.68
ZXL11	2 175.47	2 116.75
ZXL12	2 166.77	2 183.80
ZXL13	2 266.01	2 253.21
ZXL14	2 278.01	2 324.07
ZXL15	2 492.00	2 394.40
ZXL16	2 405.60	2 463.70
ZXL17	2 580.22	2 523.72
ZXL18	2 577.71	2 597.30
ZXL19	2 742.82	2 795.90

Note: GA denotes the genetic algorithm, and IMM denotes the impact matrix method.



(a)

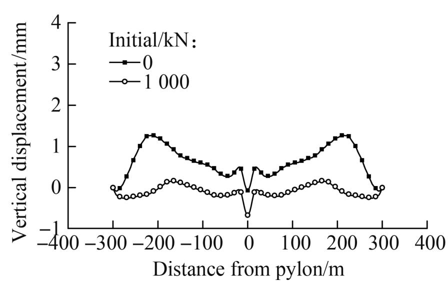


(b)

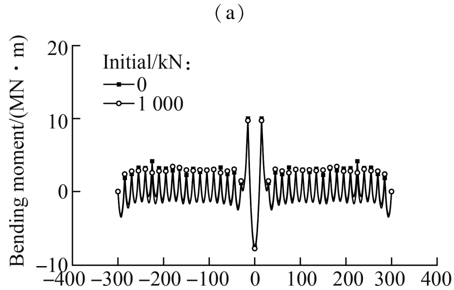
Fig. 11 Comparison of the results of two different methods. (a) Vertical bending moment of the girder; (b) Vertical displacement of the girder

advantages of fast optimization speed, high efficiency, and ideal optimization results for the influence matrix method but can also properly control the load and displacement effects of some specific structures or sections according to engineering needs, which overcomes the limitation that the influence matrix method has only a single solution.

As shown in Fig. 12, the impacts of different initial cable force values on the bridge response are considered. The initial cable force is a crucial component of the optimization process. The vertical displacement and vertical



(a)



(b)

Fig. 12 Comparison of the optimization results under different initial cable forces of the genetic algorithm. (a) Vertical displacement of the girder; (b) Vertical bending moment of the girder bending moment of the main girder under two distinct initial cable force circumstances are depicted in Figs. 12(a) and (b), respectively. The findings indicate that, with two different initial cable force values, the optimal values of the displacement and bending moment of the main beam are approximately equal. However, when the initial cable force is 0, the main beam undergoes a slight positive displacement. The impact of the initial cable force on the optimization outcomes is insignificant for the proposed optimization approach in this research.

Fig. 13 shows the effect of the main girder under self-weighting and two kinds of live load conditions. Fig. 13 (a) shows the vertical displacement diagram of the main beam under different working conditions. Fig. 13 (b) shows the axial force diagram of the main girder under different working conditions. Fig. 13(c) shows the vertical bending moment diagram of the main girder under different working conditions. The two cases are shown in Fig. 8. The results show that the displacement of Case 2 is significantly larger than the displacement of Case 1 and self-weight, which is attributed to the smaller mid-span stiffness. The maximum displacement of the main girder is 0.530 777 m. According to the cable-stayed bridge design specifications, the allowable range of the main girder displacement is $L_K/400 = 300/400 = 0.75$ m, so the main girder displacement is within the allowable range. L_K is the span of computation. Fig. 13(b) shows that the maximum axial force of the main girder is located at the intersection of the pylon and the main girder. The increase in the axial force of the beam under live load conditions is attributed to the application of the live load, which leads to an increase in the cable force and an increase in the transverse component of the cable force. Fig. 13 (c) shows a large negative bending moment on the left side of the main beam in Case 2, which needs special attention in engineering practice. The main reason is that there is no auxiliary pier or counterweight on the side span, which is caused by the lack of constraints. The displacement and load effects of the pylon are shown in Fig. 14. Given that the bridge types investigated in this study are symmetrical on both sides, the response characteristics of the main tower are relatively simple and are no longer described here.

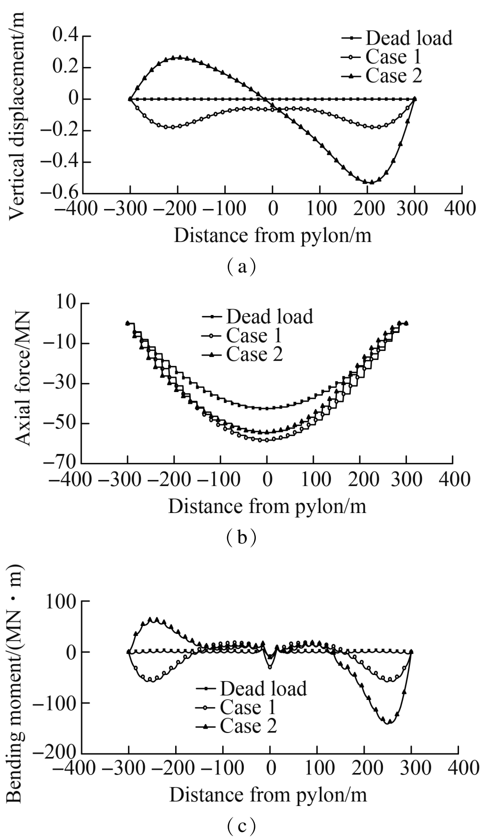


Fig. 13 Response comparison of the main girder under different load conditions. (a) Vertical displacement of the girder; (b) Longitudinal axial force diagram of the main girder; (c) Vertical bending moment of the girder

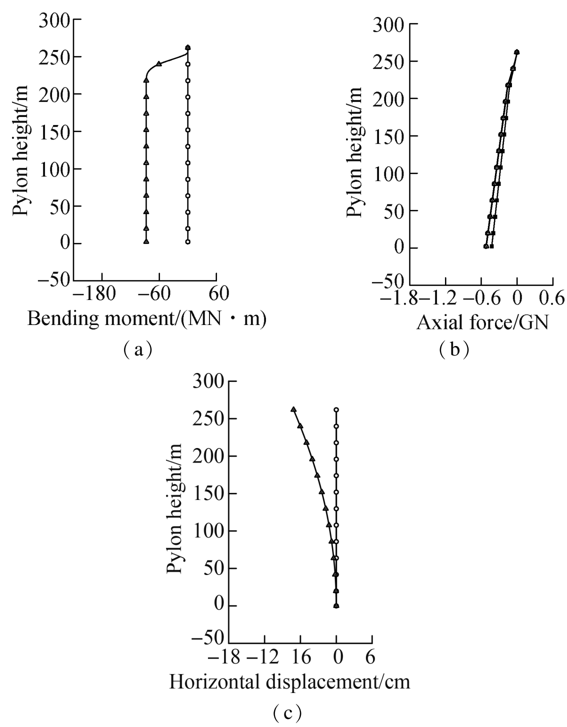


Fig. 14 Displacement load effect of the pylon under different load conditions. (a) Longitudinal bending moment diagram of the pylon; (b) Vertical axial force diagram of the pylon; (c) Longitudinal displacement diagram of the pylon

Fig. 15 shows the calculation of the normal stress of the pylon and girder under different working conditions. Fig. 15(c) shows that the main girder is in a state of full-section compression, and only the lower edge of some sections experiences tensile stress. Here, SH represents the upper edge stress of the girder, and DI represents the lower edge stress of the girder. The maximum tensile stress is 54.84 MPa, and the maximum compressive stress is 63.76 MPa. The two maximum values appear in Case 2, and the normal stress under all load conditions is within the allowable range of the main beam stress. The tensile and compressive design strengths of the girder are 260 and −260 MPa, respectively. Figs. 15(a) and (b) show that all of the sections of the pylon are in a state of full-section compression, and the maximum compressive stresses on different sides of the section are 9.38 and 9.96 MPa. The former appears in Case 1, and the latter appears in Case 2. However, both of these values are less than the compressive strength design value of 24.4 MPa for concrete. The tensile and compressive design strengths of concrete are 1.89 and −24.4 MPa, respectively.

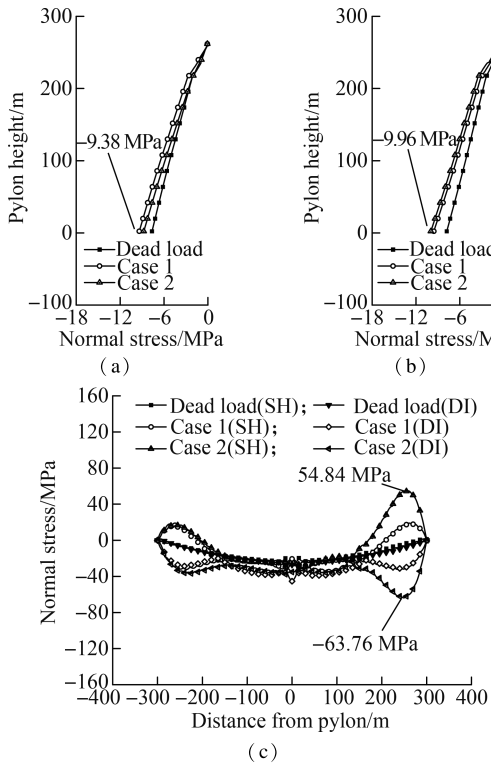


Fig. 15 Stress distributions of the pylon and girder under different load conditions. (a) Normal stress of the left pylon; (b) Normal stress of the right pylon; (c) Normal stress of the girder

5 Conclusions

1) The advantages of the influence matrix method, which simplifies a nonlinear problem to a linear problem and applies a genetic algorithm to improve the global search capability of cable force optimization, are incorporated. In the given feasible domain of a cable-stayed bridge, the proposed method can efficiently and stably conduct cable-stayed bridge cable force optimization and finally obtain the ideal initial cable force.

2) The proposed method not only has the advantages of the influence matrix method, such as fast speed, high efficiency, and ideal results but can also add specific constraints under the premise that the minimum bending moment can be the target, which can meet some specific needs in engineering practice and improve the free selectivity of the optimization results. Then, the elite retention strategy is incorporated into the optimization process. By doing so, the optimization process will achieve enhanced stability and efficiency.

3) The algorithm is insensitive to reasonable initial parameters, such as the initial value of the cable force, population size, and feasible region of the cable force. However, the initial parameters should be within a reasonable range. For large-span cable-stayed bridges, the ratio of the side span to the main span should be controlled within a reasonable range during the design process. A symmetrical span is not necessarily the most reasonable option. In some cases, auxiliary piers or counterweights should be used to increase the vertical stiffness of the bridge.

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基于影响矩阵和精英遗传算法的斜拉桥索力优化

朱　力[1]　孟博文[1]　霍学晋[2]　刘　伟[1]

([1] 北京交通大学土木建筑工程学院, 北京 100044)
([2] 中铁大桥勘察设计研究院有限公司, 武汉 430050)

摘要: 由于传统索力优化方法不能自动优化和兼顾全桥结构性能,以及与智能优化算法相结合后存在计算量大、耗时长、效率低和收敛速度慢等缺点,以 Ansys 和 Matlab 分别作为结构计算器和主控程序,以最小弯矩能为控制目标,将影响矩阵和精英保留策略引入到遗传算法中,实现成桥阶段的索力优化. 该方法能够同时兼顾主梁与主塔的受力特性,而且影响矩阵的应用可以解决每一代都需要有限元评估的问题,大大减少了计算量. 与传统的影响矩阵法相比,所提方法在吸取影响矩阵法优点的基础上,还考虑参数迭代过程,可在结构的关键截面添加特殊的约束要求,使得优化过程更加符合实际,而精英保留策略的引入可以提高进化迭代收敛的速度与进化过程的稳定性. 最后,通过一个实际工程应用验证了所提方法的可行性.

关键词: 斜拉桥;索力优化;最小弯矩能;影响矩阵;遗传算法;可行域

中图分类号: TU3